Lab Quiz 2 Solutions, Math 253 B27/B28 **Friday 02/04, 10:00**

J. Macki, D. Kinzabulatov

1.
$$\int \frac{1}{4x^2+4x+2}\,dx = \int \frac{1}{1+(2x+1)^2}\,dx = \frac{1}{2}\arctan(2x+1)+C.$$

2. (a) Simplify $\cos(\arctan(p+1))$. If we let $\theta = \arctan(p+1)$, then we can draw a right triangle with θ at a vertex, opposite side of length p+1, adjacent side of length 1. Then the hypotenuse has length $\sqrt{[p+1)^2+1} = p^2+2p+2$. Thus $\cos(\theta) = \frac{1}{p^2+2p+2}$.

   (b) Calculate

$$\int \frac{1}{2\sqrt{x}\,\sqrt{1-x}}\,dx\,\Big|_{\substack{u=\sqrt{x}\\ du=\frac{1}{2\sqrt{x}}\,dx}} = \int \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u)+C = \arcsin\left(\sqrt{x}\right)+C.$$

3. Given $f(x) = \arctan\left(\sqrt{4-x}\right)$: (a) Find the domain and range of f; (b) Show that f is one-to-one on its domain; (c) Find a formula for f^{-1}, the inverse to f.

   (a) The domain is defined by $4-x \geq 0$, i.e., $(-\infty, 4]$. The range of f is the range of $\arctan(w)$ when w is restricted to the interval $[0,\infty)$. Thus the range of f is $[0,\infty)$.

$$\text{(b)}\quad f'(x) = \frac{1}{1+(4-x)}\cdot\frac{1}{2\sqrt{4-x}}\cdot(-1) < 0$$

   on the domain of f, so f is one-to-one.

   (c) If we set $y = \arctan\left(\sqrt{4-x}\right)$ then

$$\tan(y) = \sqrt{4-x}, \quad \text{so} \quad x = 4-\tan^2(y).$$

   Interchanging x and y, we have $f^{-1}(x) = 4-\tan^2(x)$.