Solutions to Quiz 3, Math 253 B25/B26 **Wed. 02/16 , 1300 hours**

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1. Calculate (a) $$\int_0^1 \frac{1}{[x^2+9]^{\frac{3}{2}}}\,dx\Big|_{\substack{x=3\tan(u)\\ dx=3\sec^2(u)\,du}} = \frac{1}{9}\int_0^{\arctan\left(\frac{1}{3}\right)} \frac{1}{\sec^3(u)} sec^2(u)\,du =$$
$$= \frac{1}{9}\sin(u)\,\Big|_0^{\arctan\left(\frac{1}{3}\right)} = \frac{1}{9\sqrt{10}}.$$
It is o.k. to not change the limits and convert back to x at the end.

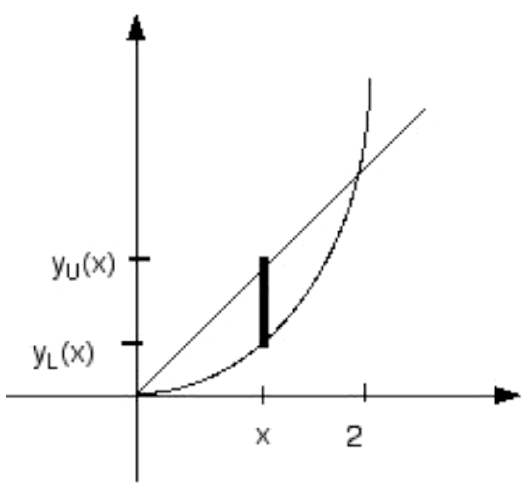
$$\text{(b)} \quad \int \frac{x}{[1-(x-2)^2]^{\frac{3}{2}}}\,dx\Big|_{\substack{x-2=\sin(u)\\ dx=\cos(u)\,du}} = \int \frac{2+\sin(u)}{\cos^3(u)}\cos(u)\,du =$$

$$2\int \sec^2(u)\,du + \int \frac{\sin(u)}{\cos^2(u)}\,du = 2\tan(u)+\sec(u)+C = 2\frac{x-2}{\sqrt{1-[x-2]^2}} + \frac{1}{\sqrt{1-[x-2]^2}} + C.$$

2. Calculate $$\int e^x \arctan\left(e^x\right)\,dx\Big|_{\substack{u=e^x\\ du=e^x dx}} = \int \arctan(u)\,du\Big|_{\substack{U=\arctan(u),\ dV=1\\ dU=\frac{1}{1+u^2}du\ ,\ V=u}} =$$
$$= u\arctan(u) - \int \frac{u}{1+u^2}\,du = e^x\arctan(e^x) - \frac{1}{2}\ln\left(1+e^{2x}\right) + C,$$
where we first used substitution, then integrated by parts.

3. Consider the finite region between the curves $y=x^3$ and $y=4x$, $0 \le x \le 2$. Set up an integral for the area of this region (a) using the x-axis, and (b) using the y-axis, as the axis of integration. DO NOT EVALUATE.

$$\text{(a)} \quad \int_0^2 \left[4x - x^3\right]\,dx \quad \text{(b)} \quad \int_0^8 \left[y^{\frac{1}{3}} - \frac{1}{4}y\right]\,dy.$$



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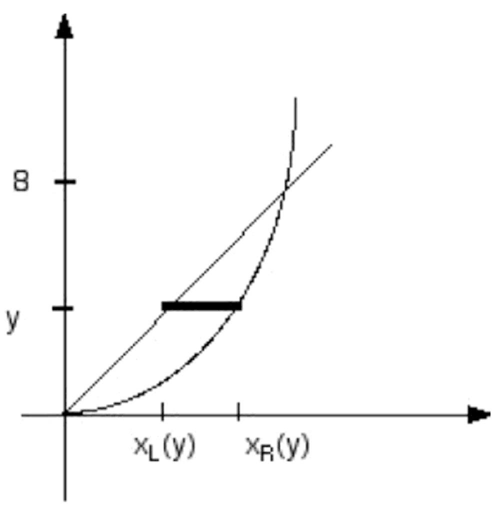

(a) $A = \int_0^2 [\, y_U(x) - y_L(x)\,]\,dx$

(b) $A = \int_0^8 [\, x_R(y) - x_L(y)\,]\,dy$