Solutions to Lab Quiz 3, Math 253 B27/B28 , Friday 02/18 , 10:00

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1. Calculate (a) $\int_0^2 \frac{x^3}{[4+x^2]^{\frac{5}{2}}}\, dx \,|_{stackrelx=2\tan(u)dx=2\sec^2(u)\,du} = \int_0^{\frac{\pi}{4}} \frac{8tan^3(u)}{[4+tan^2(u)]^{\frac{5}{2}}} \sec^2(u)\, du =$

$$= \int_0^{\frac{\pi}{4}} \frac{2^3 tan^3(u)}{2^5 \sec^3(u)}\, du = \frac{1}{4}\int_0^{\frac{\pi}{4}} \sin^3(u)\, du =$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{4}} \left[1-\cos^2(u)\right]\sin(u)\, du = \left[-\cos(u)+\frac{1}{3}\cos^3(u)\right]|_0^{\frac{\pi}{4}} = -\frac{1}{\sqrt{2}}+1+\frac{1}{3\cdot 2\sqrt{2}}-\frac{1}{3}.$$

(b) $\int \frac{1}{[1-(x-2)^2]^{\frac{3}{2}}}\, dx|_{\substack{x-2=\sin(u)\\ dx=\cos(u)\,du}} = \int \frac{1}{[1-\sin^2(u)]^{\frac{3}{2}}}\cos(u)\, du =$

$$= \int \frac{1}{\cos^2(u)}\, du = \tan(u) + C = \frac{x-2}{\sqrt{1-(x-2)^2}} + C.$$

2. Calculate $\int x \arctan(x)\, dx.$

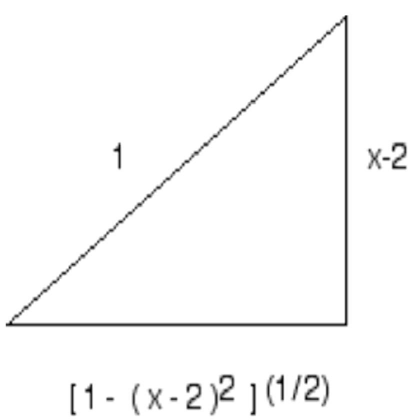
We use integration by parts, with

$$U = \arctan(x),\ dV = x,\ dU = \frac{1}{1+x^2},\ V = \frac{1}{2}x^2.$$

So the integral becomes

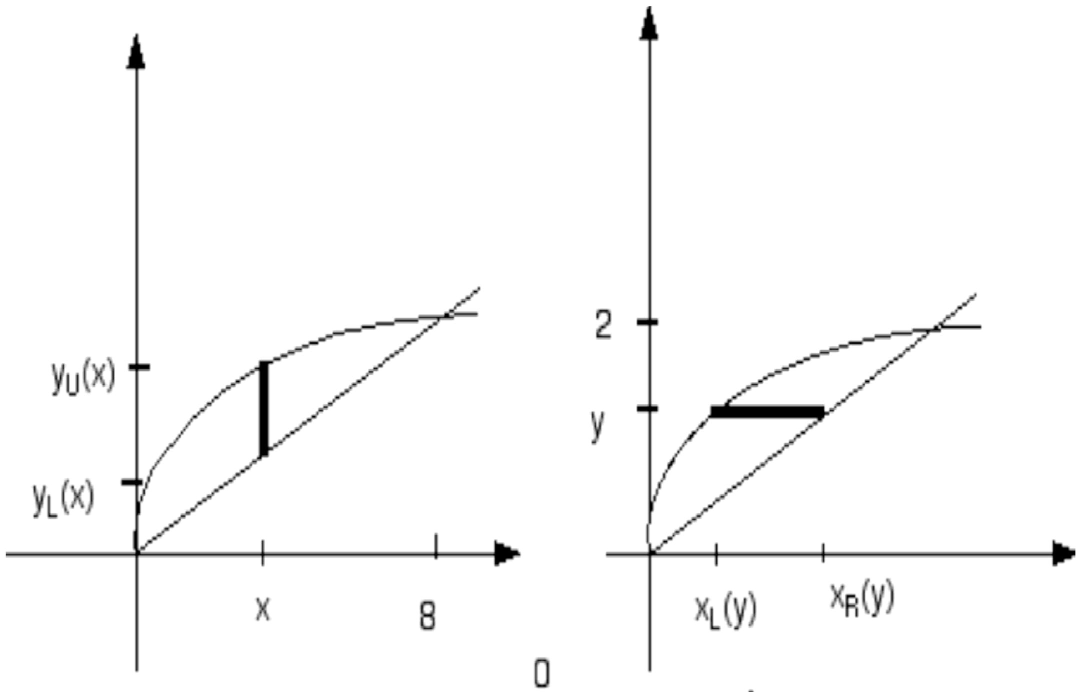
$$\frac{1}{2}x^2\arctan(x) - \frac{1}{2}\int \frac{x^2}{1+x^2}\, dx = \frac{1}{2}x^2\arctan(x) - \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\, dx =$$

$$= \frac{1}{2}x^2\arctan(x) - \frac{1}{2}x + \frac{1}{2}\arctan(x) + C.$$



3. Consider the finite region between the curves $y = x^{\frac{1}{3}}$ and $y = \frac{1}{4}x$, $0 \le x \le 8$. Set up an integral for the area of this region (a) using the x-axis, and (b) using the y-axis, as the axis of integration. DO NOT EVALUATE. (Note that the curves intersect at x =0 and x = 8 and the graph of $y = x^{\frac{1}{3}}$ lies above the straight line).

$$\text{(a)} \quad \int_0^8 \left[x^{\frac{1}{3}} - \frac{1}{4}x \right] \, dx, \qquad \text{(b)} \quad \int_0^2 \left[4y - y^3 \right] \, dy.$$



(a) $A = \int_0^4 [\, y_U(x) - y_L(x) \,]\, dx$

(b) $A = \int_0^2 [\, x_R(y) - x_L(y) \,]\, dy$