Solutions to Lab Quiz 5, Math 253 B25/B26   Wed. 04/06 , 1300 hours

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1. Use the method of discs to find the volume of the solid obtained by rotation about the x-axis of the bounded region determined by the curves:

$$y = e^x - 1, \qquad x = 1, \qquad x = 2, \qquad y = 0.$$

Solution:

$$dV = \pi R^2 dx = \pi \left\{ [e^x - 1]^2 \right\} dx \quad \Rightarrow V = \pi \int_1^2 \left[e^{2x} - 2e^x + 1 \right] dx =$$

$$= \pi \left[\frac{1}{2} e^{2x} - 2e^x + x \right] |_1^2 = \pi \left[\frac{1}{2}(e^4 - e^2) - 2(e^2 - e) + 2 - 1 \right].$$

2. Use the method of shells to set up, but do not evaluate, the integral for the volume of the solid obtained by rotation of the region from problem 1 about the y-axis.
Solution:

$$dV = 2\pi Rh\, dx = 2\pi x f(x)\, dx = 2\pi x \left[e^x - 1 \right] dx \quad \Rightarrow \quad V = 2\pi \int_1^2 x[e^x - 1]\, dx.$$

2 points extra credit: Evaluate the integral.
Solution: Use integration by parts on $\int xe^x \, dx$ with $U = x$ to get:

$$V = 2\pi \left[(x-1)e^x - \frac{1}{2}x^2 \right] |_1^2 = 2\pi \left[(2-1)e^2 - 0e^1 - \frac{1}{2}[4-1] \right].$$

3. (a) Set up, but do not evaluate, the integral for the arclength of the curve defined by:

$$y = \arcsin(x), \qquad 0 \leq x \leq 1.$$

(b) Set up, but do not evaluate, the integral for the surface area obtained by rotating the curve in (a) about the x-axis.

Solution: (a) We have

$$y = \arcsin(x), \quad y' = \frac{1}{\sqrt{1-x^2}}, \quad ds = \left[1 + \frac{1}{1-x^2} \right]^{\frac{1}{2}} dx = \frac{\sqrt{2-x^2}}{\sqrt{1-x^2}}\, dx.$$

So the arclength is

$$\int_0^1 \frac{\sqrt{2-x^2}}{\sqrt{1-x^2}}\, dx.$$

(b) We have

$$dS = 2\pi R ds = 2\pi f(x) ds = 2\pi \arcsin(x) ds,$$

so the surface area is

$$S = 2\pi \int_0^1 \arcsin(x) \frac{\sqrt{2-x^2}}{\sqrt{1-x^2}}\, dx.$$