Solutions to Lab Quiz 5, Math 253 B27/B28 **Friday 04/08 , 10:00**

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1. Consider the bounded region defined by the curves:

$$y = \ln(x), \qquad y = 0, \qquad 1 \le x \le e^3.$$

Use the method of discs to set up, but do not evaluate, the integral for the volume obtained by rotating this region about the x-axis.
Solution:

$$dV = \pi R^2\, dx = \pi[\ln(x)]^2\, dx \quad \Rightarrow \quad V = \pi \int_1^{e^3} [\ln(x)]^2\, dx.$$

2. For the region from problem 1, use the method of shells to set up, but do not evaluate, the integral for the volume obtained by rotation about the y-axis.
Solution:

$$dV = 2\pi Rh\, dx = 2\pi x \ln(x)\, dx \quad \Rightarrow \quad V = 2\pi \int_1^{e^3} x \ln(x)\, dx.$$

2 points extra credit: Evaluate the integral.
Solution: Use integration by parts with $f(x) = \ln(x)$, $g'(x) = x$:

$$V = 2\pi \left[\frac{1}{2} x^2 \ln(x) \,|_1^{e^3} - \frac{1}{2} \int_1^{e^3} x\, dx \right] = 2\pi \left[\frac{3}{2} e^6 - \frac{1}{4} x^2 \,|_1^{e^3} \right] =$$

$$= 2\pi \frac{1}{2} \left[3e^6 - \frac{1}{2} e^6 + \frac{1}{2} \right] = \frac{\pi}{2} \left[5e^6 + 1 \right].$$

3. (a) Write down, but do not evaluate, the integral for the arclength of the curve

$$y = \ln(1+x), \quad 0 \le x \le e^2.$$

(b) Write down, but do not evaluate, the integral for the surface area obtained by rotating the curve in (a) about the y-axis.
Solution: (a)

$$y'(x) = \frac{1}{1+x} \quad \Rightarrow \quad ds = \left[1 + \frac{1}{(1+x)^2} \right]^{\frac{1}{2}}.$$

Therefore the arclength is

$$s = \int_0^{e^2} \left[1 + \frac{1}{(1+x)^2} \right]^{\frac{1}{2}} dx.$$

(b) We have $d\mathcal{A} = 2\pi R\, ds = 2\pi x\, ds$, so

$$\mathcal{A} = 2\pi \int_0^{e^2} x \left[1 + \frac{1}{(1+x)^2} \right]^{\frac{1}{2}} dx.$$