Mathematics 253 Lab Worksheet 6 Solutions, Labs B25-B32

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Note: Friday labs will be writing their delayed quiz in place of this worksheet.

1. Consider the bounded region B determined by the x-axis, the vertical line $x = \frac{\pi}{2}$, and the graph of $\sin(x)$, $\frac{\pi}{2} \leq x \leq \pi$. (TAs: sketch it on the board).
   Set up the integrals for the volume of the solid resulting from:
   (a) rotating B about the x-axis, using discs.
   (b) rotating B about the line y = -2, using discs.
   (c) rotating B about the y-axis, using shells.

   Solution: (a)
$$\pi \int_{\frac{\pi}{2}}^{\pi} \sin^2(x)\, dx\,.$$
   (b)
$$\pi \int_{\frac{\pi}{2}}^{\pi} \left[(2+\sin(x))^2 - 2^2\right]\, dx$$
   (c)
$$2\pi \int_{\frac{\pi}{2}}^{\pi} x\sin(x)\, dx\,.$$

2. Find the arclength of the curve $y = \ln|\cos(x)|$, $0 \leq x \leq \frac{\pi}{4}$.

   Solution:
$$y = \ln|\cos(x)|\,, \quad y' = -\tan(x),\ \ 1 + (y')^2 = 1 + \tan^2(x) = \sec^2(x),$$
   so
$$ds = \left[1 + (y')^2\right]^{\frac{1}{2}}\, dx = \sec(x)\, dx\,.$$
   Then the arclength is
$$\int_0^{\frac{\pi}{4}} \sec(x)\, dx = \ln|\sec(x) + \tan(x)|\,|_0^{\frac{\pi}{4}} = \ln(1+\sqrt{2}).$$

3. (a) Set up the integral for the surface area obtained by rotating the curve $y = x^2$, $0 \leq x \leq 2$ about the x-axis.
   (b) Same as (a) except rotate about the y-axis.
   (c) (If you have time) Evaluate the integrals from (a) and (b).

   Solution: We need the arc length element,
$$ds = \left[1 + (2x)^2\right]^{\frac{1}{2}}\, dx\,.$$
   Then for (a) we have
$$dA = 2\pi y\, ds = 2\pi x^2 \left[1 + 4x^2\right]^{\frac{1}{2}}\, dx,\quad A = \int_0^2 2\pi x^2 \left[1+4x^2\right]^{\frac{1}{2}}\, dx\,.$$

For (b) we have

$$dA = 2\pi x \, ds = 2\pi x \left[1 + 4x^2\right]^{\frac{1}{2}} \, dx \,, \quad A = 2\pi \int_0^2 x \left[1 + 4x^2\right]^{\frac{1}{2}} \, dx \,.$$

(c) The integral in (a) is evaluated by setting $x = \frac{1}{2}\tan(u)$. The integral in (b) is evaluated by letting $u = 1 + 4x^2$..