The University of Calgary
Department of Mathematics and Statistics
MATH 253
Handout # 2-solution

A

For 1)

sin function is $2\pi-$periodic and $\sin(\theta \pm \pi) = -\sin\theta$ so

$\sin(\frac{17}{6}\pi) = -\sin(\frac{17}{6}\pi - 3\pi) = -\sin\left(\frac{17}{6}\pi - \frac{18}{6}\pi\right) = -\sin\dfrac{-\pi}{6} = \sin\dfrac{\pi}{6}$

and $\frac{\pi}{6} \in \left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right)$,and thus $\arcsin(\sin(\frac{17}{6}\pi) = \arcsin(\sin(\frac{\pi}{6})) = \dfrac{\pi}{6}$

since on $(-\frac{\pi}{2}, \frac{\pi}{2})$, the functions arcsin and sin are inverse and cancel each other.

For 2)

using Chain Rule $f'(x) = \dfrac{1}{1+\left(2\sqrt{x}-\dfrac{3}{x}\right)^2} \cdot \left(\frac{2}{2\sqrt{x}} + \frac{3}{x^2}\right) = \dfrac{\frac{1}{\sqrt{x}} + \frac{3}{x^2}}{1+4x-\frac{12}{\sqrt{x}}+\frac{9}{x^2}} \cdot \dfrac{x^2}{x^2} =$

$= \dfrac{x^{\frac{3}{2}}+3}{x^2+4x^3-12x^{\frac{3}{2}}+9}$ for the domain $x \neq 0, x \geq 0,$ arctan is differentiable everywhere,

so together $D_{f'} = (0, +\infty)$.

For 3)

First , find the intersection points between two given curves:

$f(x) = 2|x| = x + 3 = g(x),$

$\pm 2x = x + 3, x = 3,$and $x = -1$

now testing which function is the top:

$- - - -_{-1} - - - - -_{x=0} - - - -_{3} - - f(0) = 0 < g(0) = 3$

so g is the top and

$A = \int\limits_{-1}^{3} [g(x) - f(x)]\,dx = \int\limits_{-1}^{3} (x+3-2|x|)dx = \int\limits_{-1}^{3} (x+3)dx - 2\int\limits_{-1}^{3} |x|\,dx =$

$= \left[\frac{1}{2}x^2\right]_{-1}^{3} + [3x]_{-1}^{3} - 2\int\limits_{-1}^{0} |x|\,dx - 2\int\limits_{0}^{3} |x|\,dx =$ (since it is easier than integrate $|x|$) $=$

$= \frac{1}{2}(9-1) + 3(3+1) - 2\int\limits_{-1}^{0} (-x)dx - 2\int\limits_{0}^{3} xdx = 4 + 12 + [x^2]_{-1}^{0} - [x^2]_{0}^{3} = 16 - 1 - 9 = 6.$

B

For 1)

tangent function is $\pi - \textit{periodic}$ so $\tan(x + 2\pi) = \tan(x)$ and if $x \in \left(-\frac{5}{2}\pi, -\frac{3}{2}\pi\right)$,

$x + 2\pi \in \left(-\frac{1}{2}\pi, \frac{1}{2}\pi\right)$ and $\arctan(\tan(x) = \arctan(\tan(x+2\pi)) = x + 2\pi,$

since on $(-\frac{\pi}{2}, \frac{\pi}{2})$, the functions arctan and tan are inverse and cancel each other.

For 2)

For $x \neq 0$ using Chain Rule :

$$f'(x) = \frac{1}{\sqrt{1-\left(\frac{2}{x}\right)^2}} \cdot \frac{-2}{x^2} = \frac{-2\sqrt{x^2}}{x^2\sqrt{x^2-4}} = \frac{-2|x|}{x^2\sqrt{x^2-4}} = \frac{-2}{|x|\sqrt{x^2-4}},$$

also solve $x^2-4>0, (x-2)(x+2)>0$,so by testing $-_{pos}-_{-2}-_{neg}--_{2}--_{pos}-$
$x<-2$ or $x>2$
OR for domain of f' solve:
$-1<\frac{2}{x}<1$, for $x>0$, solve $\frac{2}{x}<1, 2<x$, for $x<0$, solve $-1<\frac{2}{x}, -x>2, x<-2$
$D_{f'}=(-\infty,-2)\cup(2,+\infty)$.

For 3)
the given curves are not the graphs of functions so interchange x and y
so $1-y=x^2, x+y=1$,or $y=1-x^2, y=1-x$.
Now, find the intersection points between two given curves:
$f(x)=1-x=1-x^2=g(x)$,
$x^2-x=0, x(x-1)=0, x=0$,and $x=1$

now testing which function is the top:
$\overline{}_{0}-----_{x=\frac{1}{2}}----_{1}--\ f(\frac{1}{2})=\frac{1}{2}<g(\frac{1}{2})=\frac{3}{4}$
so g is the top and

$$A=\int_0^1[g(x)-f(x)]\,dx=\int_0^1(1-x^2-1+x)dx=\int_0^1(x-x^2)dx=$$
$$=\left[\frac{1}{2}x^2\right]_0^1-\left[\frac{x^3}{3}\right]_0^1=\frac{1}{2}-\frac{1}{3}=\frac{3-2}{6}=\frac{1}{6}.$$

C
For 1)
tangent function is $\pi-\mathit{periodic}$ so $\tan(-\frac{3}{4}\pi)=\tan(-\frac{3}{4}\pi+\pi)=\tan(\frac{1}{4}\pi)$
and since $\frac{1}{4}\pi\in(-\frac{\pi}{2},\frac{\pi}{2})$, the functions arctan and tan are inverse and
cancel each other so $\arctan(\tan(-\frac{3}{4}\pi))=\arctan(\tan\frac{\pi}{4})=\dfrac{\pi}{4}$
Or $tan\frac{-3\pi}{4}=tan\frac{\pi}{4}=1$ and $\arctan 1=\dfrac{\pi}{4}$

For 2)
using Chain Rule :$f'(x)=\dfrac{1}{\sqrt{1-\left(\sqrt{x+3}-2\right)^2}}\cdot\dfrac{1}{2\sqrt{x+3}}=\dfrac{1}{2\sqrt{x+3}\sqrt{4\sqrt{x+3}-x-6}}$
for domain of f' solve:
$x+3>0$ and $-1<\sqrt{x+3}-2<1$,so $x>-3$ and $1<\sqrt{x+3}<3$,
$1<x+3<9$ and $x>-3$,thus $-2<x<6$,so $D_{f'}=(-2,6)$.

For 3)
first find the intersection points between two given curves:
$f(x)=3x-3=1-x^2=g(x)$,
$x^2+3x-4=0, (x+4)\cdot(x-1)=0, x=-4$,and $x=1$

now testing which function is the top:
$\overline{}_{-4}-----_{x=0}----_{1}--\ f(0)=-3<g(0)=1$
so g is the top and

$A = \int\limits_{-4}^{1} [g(x) - f(x)]\, dx = \int\limits_{-4}^{1} (1 - x^2 - 3x + 3) dx = \int\limits_{-4}^{1} (4 - x^2 - 3x)\, dx =$
$= 4 \cdot (1+4) - \left[\frac{x^3}{3}\right]_{-4}^{1} - \frac{3}{2}\left[x^2\right]_{-4}^{1} = 20 - \frac{1}{3} \cdot (1 + 4^3) - \frac{3}{2} \cdot (1 - 16) =$
$= 20 - \frac{65}{3} + \frac{45}{2} = \frac{125}{6}.$

D

For 1)

$\cos^2\theta + \sin^2\theta = 1 \qquad 1 + \tan^2\theta = \frac{1}{\cos^2\theta}$ so

$\cos^2\theta = \dfrac{1}{1+\tan^2\theta}$ and for $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\cos\theta = \dfrac{1}{\sqrt{1+\tan^2\theta}}$ (positive)

for any x $\quad \arctan x = \theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

so $\cos(\arctan x) = \dfrac{1}{\sqrt{1 + [\tan(\arctan x]^2}} = \dfrac{1}{\sqrt{1+x^2}}$

since tan always cancels arctan .

For 2)

using Chain Rule $f'(x) = \frac{1}{1+\left(1-\frac{2}{x^2}\right)^2} \cdot \left(\frac{4}{x^3}\right) = \dfrac{4}{x^3\left(1 + 1 - \frac{4}{x^2} + \frac{4}{x^4}\right)} \cdot \frac{x}{x} = \dfrac{4x}{2x^4 - 4x^2 + 4}$

for $x \neq 0$, arctan is differentiable everywhere,so

$D_{f'} = (-\infty, 0) \cup (0, +\infty)$.

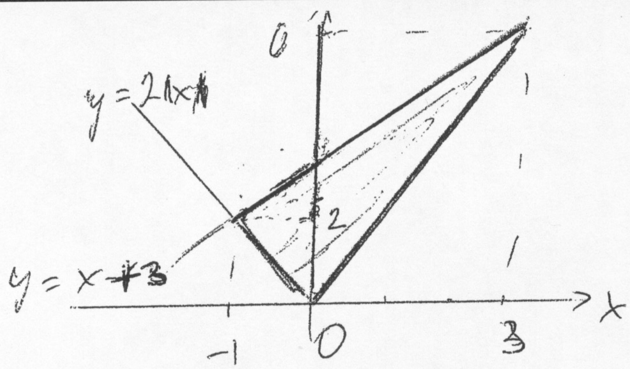
For 3)

First , find the intersection points between two given curves:
$f(x) = x^{\frac{1}{3}} = x = g(x)$,
so $x = x^3, x = 0$ and $x^2 = 1$,thus $x = \pm 1$

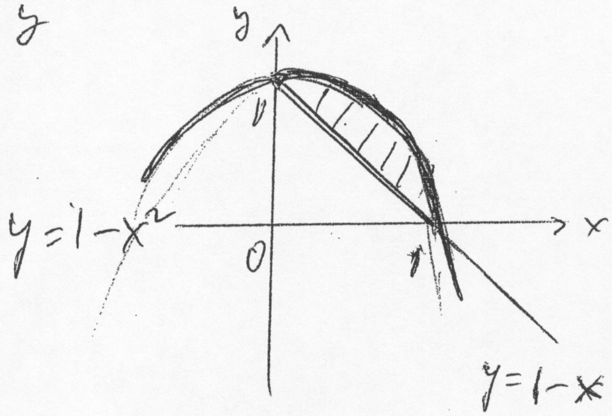
now testing which function is the top:
$\overline{\qquad}_{-1} - -_{x=-\frac{1}{8}} - - -_{0} - -_{x=\frac{1}{8}} - -_{3} - - \; f(-\frac{1}{8}) = -\frac{1}{2} < g(-\frac{1}{8}) = -\frac{1}{8}$
so g is the top on $(-1, 0)$,
but $f(\frac{1}{8}) = \frac{1}{2} > g(\frac{1}{8}) = \frac{1}{8}$ so f is the top on $(0, 1)$

$A = \int\limits_{-1}^{0} [g(x) - f(x)]\, dx + \int\limits_{0}^{1} [f(x) - g(x)]\, dx = \int\limits_{-1}^{0} (x - x^{\frac{1}{3}}) dx + \int\limits_{0}^{1} (x^{\frac{1}{3}} - x) dx =$
$= \left[\frac{1}{2}x^2\right]_{-1}^{0} - \left[\frac{3}{4}x^{\frac{4}{3}}\right]_{-1}^{0} + \left[\frac{3}{4}x^{\frac{4}{3}}\right]_{0}^{1} - \left[\frac{1}{2}x^2\right]_{-1}^{0} = -\frac{1}{2} + \frac{3}{4} + \frac{3}{4} - \frac{1}{2} = \dfrac{1}{2}.$

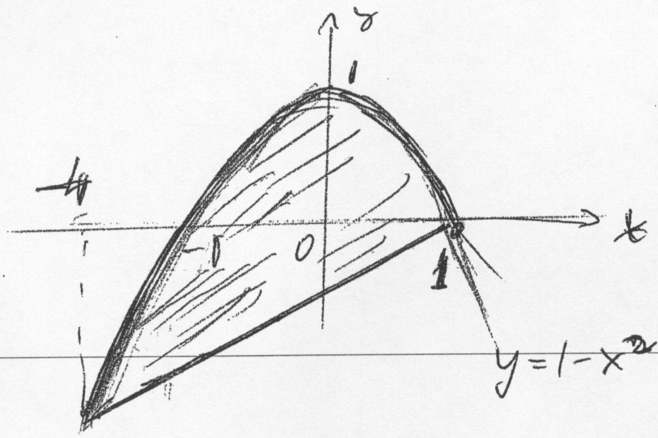
A3



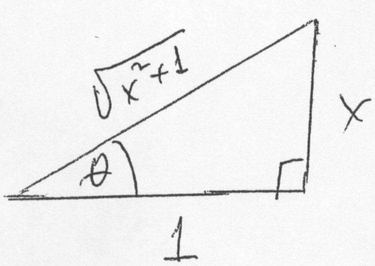
B3 x ⟺ y



C3



D1



D3