

**INTEGRATION RULES**

$$\int (Af(x) + Bg(x)) dx = A \int f(x) dx + B \int g(x) dx$$

$$\int f'(g(x))g'(x) dx = f(g(x)) + C$$

$$\int U(x) dV(x) = U(x)V(x) - \int V(x) dU(x)$$

$$\int_b^a f'(x) dx = f(b) - f(a)$$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

**ELEMENTARY INTEGRALS**

$$\int x^r dx = \frac{1}{r+1} x^{r+1} + C \text{ if } r \neq -1$$

$$\int \frac{dx}{x} = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \tan x dx = \ln|\sec x| + C$$

$$\int \cot x dx = \ln|\sin x| + C$$

$$\int \sec x dx = \ln|\sec x + \tan x| + C$$

$$\int \csc x dx = \ln|\csc x - \cot x| + C$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \frac{x}{a} + C \quad (a > 0, |x| < a)$$

$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C \quad (a > 0)$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{x+a}{x-a} \right| + C \quad (a > 0)$$

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1} \left| \frac{x}{a} \right| + C \quad (a > 0, |x| > a)$$

**TRIGONOMETRIC INTEGRALS**

$$\int \sin^2 x dx = \frac{x}{2} - \frac{1}{4} \sin 2x + C$$

$$\int \cos^2 x dx = \frac{x}{2} + \frac{1}{4} \sin 2x + C$$

$$\int \tan^2 x dx = \tan x - x + C$$

$$\int \cot^2 x dx = -\cot x - x + C$$

$$\int \sec^3 x dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$$

$$\int \csc^3 x dx = -\frac{1}{2} \csc x \cot x + \frac{1}{2} \ln |\csc x - \cot x| + C$$

$$\int \sin ax \sin bx dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \cos ax \cos bx dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \sin ax \cos bx dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} + C \text{ if } a^2 \neq b^2$$

$$\int \sin^n x dx = -\frac{1}{n} \sin^{n-1} x \cos x + \frac{n-1}{n} \int \sin^{n-2} x dx$$

$$\int \cos^n x dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2} x dx$$

$$\int \tan^n x dx = \frac{1}{n-1} \tan^{n-1} x - \int \tan^{n-2} x dx \text{ if } n \neq 1$$

$$\int \cot^n x dx = \frac{-1}{n-1} \cot^{n-1} x - \int \cot^{n-2} x dx \text{ if } n \neq 1$$

$$\int \sec^n x dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x dx \text{ if } n \neq 1$$

$$\int \csc^n x dx = \frac{-1}{n-1} \csc^{n-2} x \cot x + \frac{n-2}{n-1} \int \csc^{n-2} x dx \text{ if } n \neq 1$$

$$\int \sin^n x \cos^m x dx = -\frac{\sin^{n-1} x \cos^{m+1} x}{n+m} + \frac{n-1}{n+m} \int \sin^{n-2} x \cos^m x dx \text{ if } n \neq -m$$

$$\int \sin^n x \cos^m x dx = \frac{\sin^{n+1} x \cos^{m-1} x}{n+m} + \frac{m-1}{n+m} \int \sin^n x \cos^{m-2} x dx \text{ if } m \neq -n$$

$$\int x \sin x dx = \sin x - x \cos x + C$$

$$\int x \cos x dx = \cos x + x \sin x + C$$

$$\int x^n \sin x dx = -x^n \cos x + n \int x^{n-1} \cos x dx$$

$$\int x^n \cos x dx = x^n \sin x - n \int x^{n-1} \sin x dx$$

## TRIGONOMETRIC IDENTITIES

$$\sin^2 x + \cos^2 x = 1$$

$$\sec^2 x = 1 + \tan^2 x$$

$$\csc^2 x = 1 + \cot^2 x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$$

$$\sin(-x) = -\sin x$$

$$\sin(\pi - x) = \sin x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos(-x) = \cos x$$

$$\cos(\pi - x) = -\cos x$$

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

## QUADRATIC FORMULA

$$Ax^2 + Bx + C = 0 \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

INTEGRALS INVOLVING  $\sqrt{x^2 \pm a^2}$  ( $a > 0$ )

(If  $\sqrt{x^2 - a^2}$ , assume  $|x| > a > 0$ .)

$$\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right| + C$$

$$\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \tan^{-1} \frac{\sqrt{x^2 - a^2}}{a} + C$$

$$\int x^2 \sqrt{x^2 \pm a^2} dx = \frac{x}{8} (2x^2 \pm a^2) \sqrt{x^2 \pm a^2} - \frac{a^4}{8} \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$\int \frac{x^2}{\sqrt{x^2 \pm a^2}} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \mp \frac{a^2}{2} \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$\int \frac{\sqrt{x^2 \pm a^2}}{x^2} dx = -\frac{\sqrt{x^2 \pm a^2}}{x} + \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

$$\int \frac{dx}{x^2 \sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x} + C$$

$$\int \frac{dx}{(x^2 \pm a^2)^{3/2}} = \frac{\pm x}{a^2 \sqrt{x^2 \pm a^2}} + C$$

$$\int (x^2 \pm a^2)^{3/2} dx = \frac{x}{8} (2x^2 \pm 5a^2) \sqrt{x^2 \pm a^2} + \frac{3a^4}{8} \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C$$

INTEGRALS OF INVERSE TRIGONOMETRIC FUNCTIONS

$$\int \sin^{-1} x dx = x \sin^{-1} x + \sqrt{1 - x^2} + C$$

$$\int \tan^{-1} x dx = x \tan^{-1} x - \frac{1}{2} \ln (1 + x^2) + C$$

$$\int \sec^{-1} x dx = x \sec^{-1} x - \ln \left| x + \sqrt{x^2 - 1} \right| + C \quad (x > 1)$$

INTEGRALS INVOLVING  $\sqrt{a^2 - x^2}$  ( $a > 0, |x| < a$ )

$$\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} + C$$

$$\int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x^2 \sqrt{a^2 - x^2}} = -\frac{\sqrt{a^2 - x^2}}{a^2 x} + C$$

$$\int \frac{\sqrt{a^2 - x^2}}{x^2} dx = -\frac{\sqrt{a^2 - x^2}}{x} - \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{dx}{x \sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right| + C$$

$$\int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}} + C$$

$$\int (a^2 - x^2)^{3/2} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \frac{x}{a} + C$$

EXPONENTIAL AND LOGARITHMIC INTEGRALS

$$\int x e^x dx = (x - 1) e^x + C$$

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \ln x - \frac{x^{n+1}}{(n+1)^2} + C, \quad (n \neq -1)$$

$$\int x^n (\ln x)^m dx = \frac{x^{n+1}}{n+1} (\ln x)^m - \frac{m}{n+1} \int x^n (\ln x)^{m-1} dx$$

$(n \neq -1)$

$$\int e^{ax} \sin bx dx = \frac{e^{ax}}{a^2 + b^2} (a \sin bx - b \cos bx) + C$$

$$\int e^{ax} \cos bx dx = \frac{e^{ax}}{a^2 + b^2} (a \cos bx + b \sin bx) + C$$