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1. Calculate

$$\begin{aligned} \int \frac{e^x + 1}{e^x + x} dx \Big|_{\substack{u=e^x+x \\ du=e^x+1}} &= \int \frac{du}{u} = \\ &= \ln |u| + C = \ln |e^x + x| + C. \end{aligned}$$

2. Calculate

$$\begin{aligned} \int_0^{\frac{\pi}{2}} \sin^3(u) \cos^3(u) du &= \int_0^{\frac{\pi}{2}} \sin^3(u) \cos^2(u) \cos(u) du \Big|_{\substack{v=\sin(u) \\ dv=\cos(u)du}} = \\ &= \int_0^1 u^3 [1 - u^2] du = \left[\frac{1}{4}u^4 - \frac{1}{6}u^6 \right]_0^1 = \frac{1}{12}. \end{aligned}$$

3. Calculate

$$\begin{aligned} \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan(r) \sec^3(r) dr &= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sec^2(r) \sec(r) \tan(r) dr \Big|_{\substack{u=\sec(r) \\ du=\sec(r)\tan(r)}} = \\ &= \int_{\sqrt{2}}^{\sqrt{2}} u^2 du = \frac{1}{3}u^3 \Big|_{\sqrt{2}}^{\sqrt{2}} = 0. \end{aligned}$$

One can also note at two different points that the integrand is odd and the interval is symmetric, so the integral is zero.