Solutions to Lab Quiz 1, Math 253 B31/B32, Friday 1/21, 1300 hours

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1. Calculate

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\, dx\Big|_{\substack{u=e^x-e^{-x} \\ du=[e^x+e^{-x}]dx}} =$$

$$= \int \frac{1}{u}\, du = \ln|u| + C = \ln[e^x - e^{-x}] + C.$$

2. Calculate

$$\int_0^{\frac{\pi}{2}} \sin^4(u)\, du = \int_0^{\frac{\pi}{2}} [\sin^2(u)]^2\, du = \int_0^{\frac{\pi}{2}} \left[\frac{1-\cos(2u)}{2}\right]^2 du =$$

$$= \int_0^{\frac{\pi}{2}} \left[\frac{1 - 2\cos(2u) + \cos^2(2u)}{4}\right] du =$$

$$= \frac{1}{4}[u - \sin(2u)]|_0^{\frac{\pi}{2}} + \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{1+\cos(4u)}{2}\, du =$$

$$= \frac{\pi}{8} + \frac{1}{2}\left[u + \frac{\sin(4u)}{4}\right]|_0^{\frac{\pi}{2}} = \frac{\pi}{8} + \frac{\pi}{4}.$$

3. Calculate $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^3(r)\, \sec(r)\, dr = 0$, since the integrand is odd and the interval is symmetric. However it is o.k. to carry out the calculation in the standard way:

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \tan^2(r)\ \tan(r)\sec(r)dr =$$

$$= \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [\sec^2(r) - 1]\ \tan(r)\sec(r)dr\Big|_{\substack{u=\sec(r) \\ du=\sec(r)\tan(r)dr}} =$$

$$= \int_{\sqrt{2}}^{\sqrt{2}} [u^2 - 1]\, du = [\frac{u^3}{3} - u]|_{\sqrt{2}}^{\sqrt{2}} = 0.$$

Note that $\sec(x) = \frac{1}{\cos(x)}$ is an even function, so we end up with $\int_{\sqrt{2}}^{\sqrt{2}}$ which is automatically zero, since the upper and lower limits are the same.