Lab Quiz 2 Solutions, Math 253 B29/B30   Thursday 02/03, 1600 hours

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1. $$\int \frac{1}{x^2+6x+10}\,dx = \int \frac{1}{1+(x+3)^2}\,dx = \arctan(x+3)+C.$$

2.(a) Simplify $\sin(\arctan(5))$. If we define $\theta = \arctan(5)$, then we can draw a right triangle with θ at a vertex, side opposite of 5 and side adjacent of 1. Then the hypotenuse has length $\sqrt{26}$, and the $\sin(\theta) = \frac{5}{\sqrt{26}}$.

(b) Evaluate

$$\int_1^e \frac{1}{x\sqrt{1-[\ln(x)]^2}}\,dx \,\Big|_{\substack{u=\ln(x)\\ du=\frac{1}{x},dx}} =$$

$$= \int_0^1 \frac{1}{\sqrt{1-u^2}}\,du = \arcsin(u)\,|_0^1 = \frac{\pi}{2}.$$

Students need not change limits if they switch back to x at the end. They should be able to see that the value is $\frac{\pi}{2}$, if not take off one point.

3. (a) Given $f(x) = \arcsin(x+2)$, find the domain and range of f, and show f is one-to-one on its domain. Find a formula for f^{-1}.

(b) Same as (a), but now the function is $g(x) = \ln(\arcsin(x+2))$.

(a) The domain of $\arcsin(u)$ is $-1 \le u \le +1$, so we need to restrict x by $-1 \le x+2 \le +1$. Thus the domain of f is $[-3,-1]$. The range is the full range of arcsin, i.e., $[-\frac{\pi}{2}, \frac{\pi}{2}]$. The derivative of f is

$$f'(x) = \frac{1}{\sqrt{1-(x+2)^2}} > 0,$$

so f is one-to-one. If we set $y = \arcsin(x+2)$, then $x = -2+\sin(y)$, so $f^{-1}(x) = -2+\sin(x)$.

(b) We now must have $\arcsin(x+2) > 0$ in order to compute the ln. This means that we must restrict x so that $0 < x+2 \le +1$. Thus the domain of g is $(-2,-1]$. The range of g is the corresponding range of $\ln(\arcsin(u))$ when $0 < u \le +1$, i.e., the range of $\ln(w)$ when $0 < w \le \frac{\pi}{2}$. Thus the range of g is the interval $(-\infty, \ln(\frac{\pi}{2})]$.

If we set $y = \ln(\arcsin(x+2))$, then

$$e^y = \arcsin(x+2), \text{ so } \sin(e^y) = x+2, \text{ and} x = -2+\sin(e^y).$$

Interchanging x and y, we have

$$g^{-1}(x) = -2+\sin(e^x).$$