Solutions to Lab Quiz 3, Math 253 B29/B30 , Thursday 02/17 , 1600 hours

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1. Calculate (a) $\displaystyle\int_0^1 \frac{x^2}{[9-x^2]^{\frac{3}{2}}}\,dx\Big|_{\substack{x=3\sin(u)\\ dx=3\cos(u)\,du}} = \int_0^{\arcsin\left(\frac{1}{3}\right)} \frac{\sin^2(u)}{27\cos^3(u)}\cos(u)\,du =$

$$= \int_0^{\arcsin\left(\frac{1}{3}\right)} \tan^2(u)\,du = \int_0^{\arcsin\left(\frac{1}{3}\right)} \left[\sec^2(u)-1\right]\,du = \tan(u) - u\,\Big|_0^{\arcsin\left(\frac{1}{3}\right)} = \frac{1}{\sqrt{8}} - \arcsin\left(\frac{1}{3}\right).$$

$$\text{(b)} \quad \int \frac{1}{[1+(x-2)^2]^{\frac{3}{2}}}\,dx\,\Big|_{\substack{x-2=\tan(u)\\ dx=\sec^2(u)\,du}} =$$

$$= \int \frac{1}{\sec^3(u)}\sec^2(u)\,du = \int \cos(u)\,du = \sin(u) + C = \frac{x-2}{[1+(x-2)^2]^{\frac{1}{2}}}.$$

2. Calculate $\displaystyle\int \frac{\arcsin(\ln(x))}{x}\,dx\Big|_{\substack{u=\ln(x)\\ du=\frac{1}{x}\,dx}} = \int \arcsin(u)\,du.$

We integrate by parts, setting

$$U = \arcsin(u),\ dV = 1,\ dU = \frac{1}{\sqrt{1-u^2}},\ V = u,$$

so the integral becomes

$$UV - \int V\,dU = u\arcsin(u) - \int \frac{u}{\sqrt{1-u^2}}\,du = u\arcsin(u) + \sqrt{1-u^2} + C =$$

$$\ln(x)\arcsin(\ln(x)) + \sqrt{1-\ln^2(x)} + C.$$

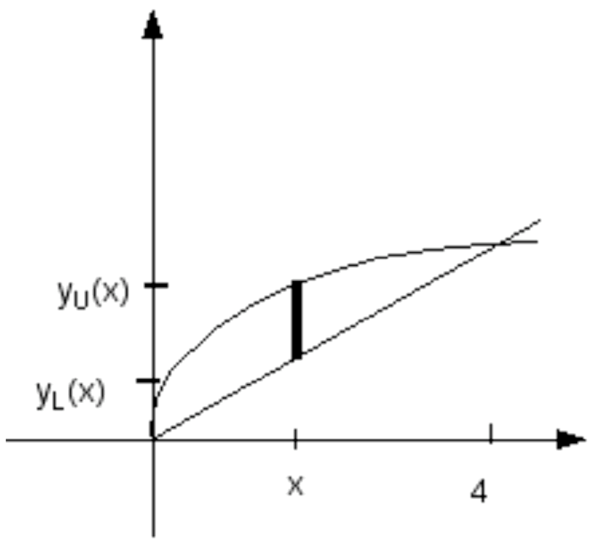
3. Consider the finite region between the curves $y=\sqrt{x}$ and $y=\frac{1}{2}x$, $0 \le x \le 4$. Set up an integral for the area of this region (a) using the x-axis, and (b) using the y-axis, as the axis of integration. DO NOT EVALUATE.

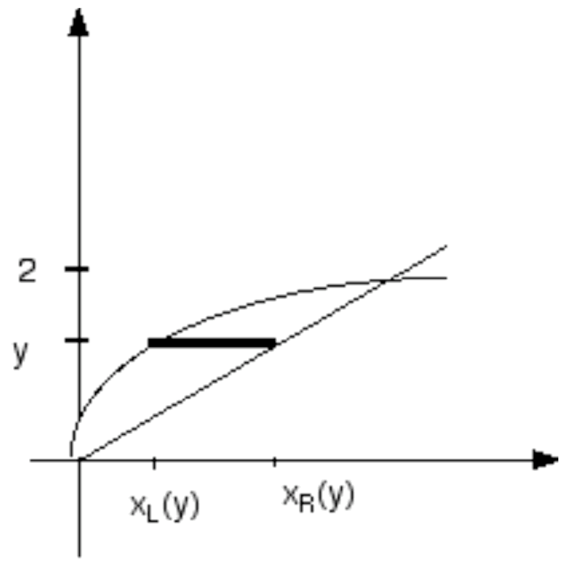
Solution: (a)

$$\int_0^4 [y_U(x) - y_L(x)]\,dx = \int_0^4 [\sqrt{x} - \frac{1}{2}x]\,dx.$$

(b)

$$\int_0^2 [x_R(y) - x_{L(y)}]\,dy = \int_0^4 [2y - y^2]\,dx.$$





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(a) $A = \int_0^4 [\, y_U(x) - y_L(x) \,]\, dx$

(b) $A = \int_0^2 [\, x_R(y) - x_L(y) \,]\, dy$