**Solutions to Lab Quiz 3, Math 253 B31/B32** **Friday 02/18 , 1300 hours**

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1. Calculate (a) $\displaystyle\int_2^3 \frac{x}{[1-(x-2)^2]^{\frac{3}{2}}}\,dx$ (b) $\displaystyle\int \frac{x^2}{[4+x^2]^{\frac{3}{2}}}\,dx.$

Solution: (a) Use the substitution $x-2=\sin(u),\ dx=\cos(u)\,du$:

$$= \int_0^{\frac{\pi}{2}} \frac{2+\sin(u)}{\left[1-\sin^2(u)\right]^{\frac{3}{2}}}\cos(u)\,du = \int_0^{\frac{\pi}{2}} \frac{2+\sin(u)}{\cos^2(u)}\,du =$$

$$= \int_0^{\frac{\pi}{2}} [2\sec^2(u) + \frac{\sin(u)}{\cos^2(u)}\,du = [2\tan(u)+\sec(u)]\ |_0^{\frac{\pi}{2}} = +\infty.$$

Full marks for proper set-up without worrying about limits (my error in assigning an improper integral).

(b) Let $x=2\tan(u),\ dx=2\sec^2(u)\,du$:

$$= \int \frac{4\tan^2(u)}{[4+4\tan^2(u)]^{\frac{3}{2}}}2\sec^2(u)\,du = \int \frac{\tan^2(u)}{\sec(u)}\,du =$$

$$= \int \frac{\sin^2}{\cos(u)}\,du = \int \frac{1-\cos^2(u)}{\cos(u)}\,du =$$

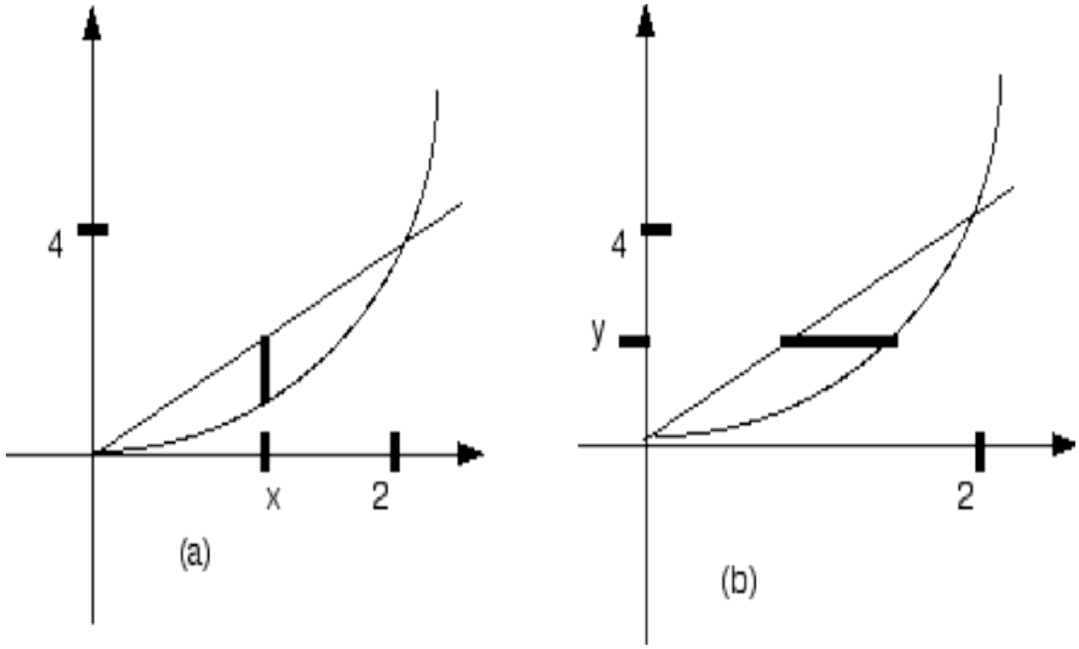
$$= \ln|\sec(u)+\tan(u)| - \sin(u) + C = \ln|\frac{x+\sqrt{x^2+4}}{2}| - \frac{x}{\sqrt{x^2+4}} + C.$$

Full marks for set-up and correct substitutions; I had promised the students that they would never have to memorize $\int \sec(u)\,du$.

2. Calculate $\int 2x\arcsin(x^2)\,dx$. First substitute $u=x^2,\ du=2x\,dx$, to get $\int \arcsin(u)\,du$, then use integration by parts with $U=\arcsin(u),\ dV=1,\ dU=\frac{1}{\sqrt{1-x^2}},\ V=x$ to get

$$= x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}} = x\arcsin(x) + \sqrt{1-x^2} + C.$$

3. Consider the finite region between the curves $y=x^2$ and $y=2x,\ \ 0\le x\le 2$. Set up an integral for the area of this region (a) using the x-axis, and (b) using the y-axis, as the axis of integration. DO NOT EVALUATE.




$$\text{(a)} \quad \int_0^2 \left[2x - x^2\right] \, dx, \quad \text{(b)} \quad \int_0^4 \left[\sqrt{y} - \frac{1}{2}y\right] \, dy.$$