Lab Quiz 4, Math 253 B29/B30 **Thursday 02/24 , 1600 hours**

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1. Explain why the following integrals are improper. Decide (with justification) whether divergent or convergent. If convergent, evaluate:

$$\text{(a)} \int_0^2 \frac{1+\sqrt{x}}{\sqrt{x}}\,dx, \quad \text{(b)} \int_4^\infty \sin(x)\,dx.$$

Solution: (a) The integral is improper because the integrand has a vertical asymptote (blows up) at $x = 0$. We back off from $x = 0$ and try to evaluate the resulting integral, and then (if we can evaluate), take a limit:

$$\lim_{A\to 0+} \int_A^2 \frac{1+\sqrt{x}}{\sqrt{x}}\,dx = \lim_{A\to 0+} \left[\int_A^2 \frac{1}{\sqrt{x}}\,dx + \int_A^2 1\,dx\right] =$$
$$= \lim_{A\to 0+} \left[2\sqrt{x}\,|_A^2 + x\,|_A^2\right] = \lim_{A\to 0+} \left[2\sqrt{2} - 2\sqrt{A} + 2 - A\right] = 2\sqrt{2} + 2.$$

Converges, to the value calculated.

(b) The integral is improper because the interval of integration is infinite. We calculate:

$$= \lim_{T\to\infty} \int_4^T \sin(x)\,dx = \lim_{T\to\infty} \left[-\cos(T) + \cos(4)\right].$$

This limit does not exist, since $\cos(T)$ keeps oscillating between -1 and $+1$ as $T \to \infty$. Divergent

2. Use comparison to decide whether or not the integral converges:

$$\int_1^\infty \frac{\cos^2(x)}{x^2}\,dx.$$

Solution: We know that $0leq \cos^2(x) \leq +1$. Dividing these inequalities by x^2 (which is positive on the domain of integration) we get

$$0 \leq \frac{\cos^2(x)}{x^2} \leq \frac{1}{x^2}.$$

The integral of the largest function, $\int_1^\infty \frac{1}{x^2}\,dx$ converges (can show directly or quote the p-integrals theorem on p. 381). Therefore our integral converges.

3. Use the trapezoidal rule with n=3 to approximate the integral:

$$\int_1^4 \frac{1}{x+1}\,dx.$$

Calculate the integral exactly, and compare your results.

Solution: The exact value is

$$\ln(x+1)|_1^4 = \ln(5) - \ln(2) = \ln(2.5) = 0.9162907319$$

The Trapezoidal Rule for this function with $n = 3$, and $a = 1,\ b = 4$ is

$$\frac{b-a}{n}\left[\frac{1}{2}f(1) + f(2) + f(3) + \frac{1}{2}f(4)\right] = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{2}\cdot\frac{1}{5} = 0.9333333333$$