Solutions to Lab Quiz 5, Math 253 B29/B30 **Thursday 04/07 , 1600 hours**

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1. Use the method of discs to find the volume of the solid obtained by rotation about the x-axis of the bounded region defined by the curves:

$$y = e^{3x}, \quad x = 1, \quad x = 3, \quad y = 0.$$

Solution: The element of volume is a (solid) disc of volume

$$dV = \pi R^2\, dx = \pi[e^{3x}]^2\, dx,$$

so the volume of the solid of revolution is

$$V = \pi \int_1^3 e^{6x}\, dx = \frac{\pi}{6} e^{6x} \,|_1^3 = \frac{\pi}{6}[e^{18} - e^6].$$

2. Use the method of shells to set up, but do not evalute, the integral for the volume of the solid obtained by rotating the region from problem 1 about the y-axis.
Solution: The element of volume is a shell of volume

$$dV = 2\pi Rh\, dx = 2\pi x e^{3x}\, dx,$$

so the volume of the solid is

$$V = 2\pi \int_1^3 x e^{3x}\, dx.$$

2 points extra credit: Evaluate the integral.
Solution: Integrate by parts, setting $f(x) = x, g'(x) = e^{3x}$:

$$V = 2\pi \left[\frac{1}{3} x e^{3x} - \frac{1}{9} e^{3x}\right] |_1^3 = 2\pi \left[\frac{1}{3}\left(3e^9 - e^3\right) - \frac{1}{9}\left(e^9 - e^3\right)\right].$$

3. (a) Write down the integral, but do not evaluate it, for the arclength of the curve

$$y = \ln\left(\sin(x^2)\right), \quad \frac{\pi}{4} \le x \le \frac{\pi}{2}.$$

(b) Write down the integral, but do not evaluate it, for the surface area obtained by rotating the curve in (a) about the x-axis.
Solution: (a) We have

$$y'(x) = 2x\cot(x^2), \qquad 1 + [y'(x)]^2 = 1 + 4x^2\cot^2(x^2),$$

so

$$ds = \left[1 + 4x^2\cot^2(x^2)\right]^{\frac{1}{2}}\, dx, \qquad s = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left[1 + 4x^2\cot^2(x^2)\right]^{\frac{1}{2}}\, dx.$$

Solution: (b) The element of surface area is

$$d\mathcal{S} = 2\pi R\, ds = 2\pi \ln\left(\sin(x^2)\right)\left[1 + 4x^2\cot^2(x^2)\right]^{\frac{1}{2}}\, dx,$$

so

$$\mathcal{S} = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\pi R\, ds = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 2\pi \ln\left(\sin(x^2)\right)\left[1 + 4x^2\cot^2(x^2)\right]^{\frac{1}{2}}\, dx,$$