Solutions to Lab Quiz 5, Math 253 B31/B32 **Friday 04/08 , 1300 hours**

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1. Consider the bounded region determined by the curves

$$y = \cos(x), \qquad y = 0, \qquad \frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}\,.$$

Use the method of discs to set up, but do not evaluate, the integral for the volume obtained by rotating this region about the x-axis.
Solution: We have

$$dV = \pi R^2\, dx = \pi \cos^2(x)\, dx \quad \Rightarrow \quad V = \pi \int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \cos^2(x)\, dx.$$

2. Use the method of shells to set up, but do not evaluate, the integral for the volume obtained by rotating the region in (a) about the y-axis.
Solution: We have

$$dV = 2\pi Rh\, dx = 2\pi x \cos(x)\, dx \quad \Rightarrow \quad V = 2\pi \int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} x \cos(x)\, dx\,.$$

2 points extra credit: Evaluate the integral.
Solution: Use integration by parts with $f(x) = x$, $g'(x) = \cos(x)$:

$$V = x\sin(x)\,\Big|_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} - \int_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} \sin(x)\, dx = \frac{5\pi}{2}\cdot(+1) - \frac{3\pi}{2}\cdot(-1) - \cos(x)\,\Big|_{\frac{3\pi}{2}}^{\frac{5\pi}{2}} = \frac{8\pi}{2}\,.$$

3. (a) Write down, but do not evaluate, the integral for the arclength of the curve

$$y = \arcsin(2x), \quad 0 \leq x \leq \frac{1}{4}\,.$$

(b) Write down, but do not evaluate, the integral for the surface area obtained by rotating the curve in (a) about the x-axis.
Solution: (a) We have

$$y'(x) = \frac{2}{\sqrt{1-4x^2}} \qquad \Rightarrow \qquad ds = \left[1 + \frac{4}{1-4x^2}\right]^{\frac{1}{2}} dx = \left[\frac{5-4x^2}{1-4x^2}\right]^{\frac{1}{2}} dx\,.$$

Then the arclength is

$$s = \int_0^{\frac{1}{4}} \left[\frac{5-4x^2}{1-4x^2}\right]^{\frac{1}{2}} dx\,.$$

(b) We have $d\mathcal{A} = 2\pi R\, ds = 2\pi \arcsin(2x)\, ds$, so the surface area is

$$\mathcal{A} = 2\pi \int_0^{\frac{1}{4}} \arcsin(2x) \left[\frac{5-4x^2}{1-4x^2}\right]^{\frac{1}{2}} dx\,.$$