Mathematics 253 Lab Worksheet, Labs B25-B32
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1. Explain why the integral is improper. Decide whether convergent or divergent–justify your conclusion.
   If convergent, evaluate:

$$(a) \quad \int_{-\infty}^{0} \frac{1}{\sqrt{1-x}}\, dx, \quad (b) \quad \int_{0}^{+\infty} \sin(x)\, dx.$$

   Solution: (a) Improper because of the $-\infty$. We calculate

$$= \lim_{T\to-\infty} \left[(-2)(1-x)^{\frac{1}{2}} \,|_T^0\right] = \lim_{T\to-\infty} \left[(-2)(1-T)^{\frac{1}{2}} + (-2)\right] = -\infty.$$

   Divergent

   (b) Improper because of the $+\infty$. We calculate

$$= \lim_{T\to+\infty} \left[-\cos(x)\,|_0^T\right] = \lim_{T\to+\infty} \left[-\cos(T)+1\right].$$

   This limit does not exist. Diverges.

2. Use comparison or analysis of asymptotic behaviour to decide whether convergent or divergent.

$$(a) \quad \int_{1}^{+\infty} \frac{\arctan(x)}{x^{\frac{7}{6}}}, \quad (b) \quad \int_{0}^{\frac{\pi}{2}} \frac{\cos(x)}{x^{\frac{1}{2}}}, \quad (c) \quad \int_{10}^{+\infty} \frac{[x^3+3x^2-2x+4]^{\frac{2}{3}}}{[5x^6+3x^3+1]^{\frac{1}{2}}}.$$

   Solution: (a) Improper, interval is infinite. Integrand is positive valued and bounded above on $[1,\infty)$:

$$\frac{\arctan(x)}{x^{\frac{7}{6}}} < \frac{\frac{\pi}{2}}{x^{\frac{7}{6}}},$$

   and the integral of the upper bound converges. Convergent.

   (b) Improper, integrand is singular at $x=0$. Excluding the singular point, on the interval of integration, the integrand is positive and we have an upper bound:

$$\frac{\cos(x)}{x^{\frac{1}{2}}} \le \frac{1}{x^{\frac{1}{2}}},$$

   also the integral of the upper bound converges. Convergent.

   (c) As $x \to +\infty$, the integrand behaves like

$$\frac{x^2}{\sqrt{5x^3}} = \frac{1}{\sqrt{5x}},$$

   the integral of which diverges, so the original integral is divergent.