Mathematics 253 Lab Worksheet, Labs B25-B32

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1. Decide whether convergent or divergent–justify your conclusion.
If convergent, evaluate:

$$\text{(a)} \int_0^1 \frac{1}{1-x^2}\,dx, \quad \text{(b)} \int_0^1 \frac{2x}{1-x^2}\,dx, \quad \text{(c)} \int_{10}^{\infty} \frac{1}{x^2-1}\,dx.$$

Solution: (a) Use partial fractions to write

$$= \lim_{a\to 1-} \int_0^a \left[\frac{\frac{1}{2}}{1+x} + \frac{\frac{1}{2}}{1-x}\right] dx = \lim_{a\to 1-} \left[\frac{1}{2}\ln|1+x| - \frac{1}{2}\ln|1-x|\right] |_0^a =$$

$$= \frac{1}{2}\lim_{a\to 1-} \left[\ln|1+a| - \ln|1-a|\right] = +\infty.$$

Diverges

$$\text{(b)} \quad = \lim_{a\to 1-} \left[-\ln|1-x^2|\,|_0^a\right] = \lim_{a\to 1-} \left[-\ln|1-a^2|\right] = +\infty.$$

Diverges

$$\text{(c)} \quad = \lim_{T\to\infty} \int_{10}^T \left[\frac{\frac{1}{2}}{x-1} - \frac{\frac{1}{2}}{x+1}\right] dx = \lim_{T\to\infty} \frac{1}{2}\left\{[\ln|x-1| - \ln|x+1|]\,|_{10}^T\right\} =$$

$$\frac{1}{2}\lim_{T\to\infty}\left\{\ln\left[\frac{T-1}{T+1}\right] - \ln\left[\frac{9}{11}\right]\right\} = \frac{1}{2}[\ln(1) - \ln\left[\frac{9}{11}\right].$$

Converges to $\frac{1}{2}\ln\left[\frac{11}{9}\right]$.