

**MATH 253**  
**Old Final Exam**  
**Answers**

1.

(a)

$$\int_0^{\infty} 3x^2 e^{x^3} dx = 1$$

(b)

$$\int x^2 e^{2x} dx = \frac{1}{2} e^{2x} \left( x^2 - x + \frac{1}{2} \right) + C$$

(c)

$$\int \frac{1}{(4-x^2)^{3/2}} dx = \frac{1}{4} \frac{x}{\sqrt{4-x^2}} + C$$

(d)

$$\int \frac{x^2}{x+1} = \frac{1}{2} x^2 - x + \ln|x+1| + C$$

(e)

$$\int \frac{1}{x^3 + 3x^2 + 2x} dx = \frac{1}{2} \ln|x| - \ln|x+1| + \frac{1}{2} \ln|x+2| + C$$

(f)

$$\int \frac{1}{x(x+1)^2} dx = \ln|x| - \ln|x+1| + \frac{1}{x+1} + C$$

(g)  $\frac{9}{2}$

(h)  $\frac{3}{4}$

(i)  $y_p(x) = \sin(x)$

2.  $8\pi$

3.  $\frac{104}{15}\pi$

4.  $y(x) = \left(\frac{3}{2}x^2 + C\right)^{1/3}$

5.  $y(x) = \frac{1}{2}x^3 + \frac{3}{2}x$

6.  $y(x) = C_1e^x + C_2e^{2x} + \frac{1}{2}x + \frac{3}{4}$

7.  $y(x) = e^{-x} + 3xe^{-x}$