MATHEMATICS 271 L01 FALL 2004
ASSIGNMENT 2

Due at 11:00 am on Friday, October 8. Your assignment must be handed in at the beginning of the lecture on September 24. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Two integers a and b are said to be *relatively prime* if, and only if $\gcd(a.b) = 1$. Thus, two integers a and b are relatively prime if, and only if there are integers x and y such that $xa + yb = 1$. Prove or disprove the following:

(a) For all integers a and b, if a and b are relatively prime then $a+b$ and $a-b$ are relatively prime.

(b) For all integers a and b, if $a+b$ and $a-b$ are relatively prime then a and b are relatively prime.

(c) For all integers a, b and c, if a and b are relatively prime, and $a \mid bc$ then $a \mid c$.

(d) For all integers a, b and c, if a and b are relatively prime, and $a \mid c$ and $b \mid c$ then $ab \mid c$.

2. Let n be a positive integer. Prove the following statements:

(a) For all integers a and b, $(a+b) \bmod n = (a \bmod n + b \bmod n) \bmod n$.

(b) For all integers a and b, $(ab) \bmod n = ((a \bmod n)(b \bmod n)) \bmod n$.

(c) For all integers a, if a and n are relatively prime then there is an integer b such that $1 \le b \le n-1$ and $(ab) \bmod n = 1$.

(d) For all integers a, x and y, if a and n are relatively prime, and $(ax) \bmod n = (ay) \bmod n$ then $x \bmod n = y \bmod n$.

3. Prove or disprove the following statements:

(a) For real numbers x and y, $\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$.

(b) For real numbers x and y, if $x + \lceil x \rceil = y + \lceil y \rceil$ then $x = y$.

(c) For real numbers y, there is a real number x such that $y = x + \lceil x \rceil$.

(d) For real numbers x and y, if $x + \lceil x \rceil = y$ then $x = y - \frac{1}{2}\lceil y \rceil$.