MATHEMATICS 271 L01 FALL 2004
ASSIGNMENT 4

Due at 11:00 am on Monday, November 16. Your assignment must be handed in at the beginning of the lecture on September 24. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Let $h : \mathbb{Z} \to \mathbb{Z}$ be the function defined by $h(x) = 2x + 1$ for all $x \in \mathbb{Z}$.

(a) Prove that h is one-to-one but not onto $\mathbb{Z}$.

(b) Find a function $g : \mathbb{Z} \to \mathbb{Z}$ so that $g \circ h$ is onto $\mathbb{Z}$.

(c) For positive integers n, define the functions $h^n : \mathbb{Z} \to \mathbb{Z}$ by putting $h^1 = h$ and for integers $k \geq 2$, $h^k = h \circ h^{k-1}$. Compute $h^2(x)$, $h^3(x)$, $h^4(x)$, and $h^5(x)$. From these, guess an explicit formula for $h^n(x)$ for all integers $n \geq 1$.

(d) Prove by induction on n that your guess in part (c) is correct.

2. Let $f : X \to Y$ and $g : Y \to Z$ be functions. Prove or disprove the following statements:

(a) If both f and g are onto then $g \circ f$ is onto.

(b) If $g \circ f$ is onto then f is onto.

(c) If $g \circ f$ is onto then g is onto.

(d) If $g \circ f$ is onto and g is one-to-one then f is onto.

3. Let $A = \{1, 2, 3, 4\}$. You must provide a proof, or an explanation to support your answer to each of the following questions.

(a) Is it true that if $h : A \to A$ is a function so that $h \circ h(1) = 2$ then h must be onto?

(b) Is there a function $g : A \to A$ so that $g \circ g(1) = 2$ and g is onto?.

(c) How many functions $f : A \to A$ are there so that $f \circ f(1) = 2$?

(d) Is it true that if $h : A \to A$ is a function so that $h \circ h = h$ then h must be onto?

(e) Is there a function $g : A \to A$ so that $g \circ g = g$, g is onto and $g(x) \neq x$ for all $x \in A$?

(e) How many functions $f : A \to A$ are there so that $f \circ f = f$, f is onto and $f(x) \neq x$ for all $x \in A$?