MATHEMATICS 271 L01 FALL 2004
QUIZ 1
Monday, September 20, 2004 **Duration: 30 minutes.**

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[3] **1**. Write the *negation* (in good English) of each of the following statements. The answer " It is not the case that ..." is **not** acceptable.
(a) For all real numbers x and y, if x is rational and y is irrational then $x+y$ is irrational and xy is irrational.
Solution: There exist real numbers x and y such that x is rational and y is irrational, but $x+y$ is rational or xy is rational.
(b) The sum of two irrational numbers is irrational.
Solution: There are two irrational numbers whose sum is rational.
(c) There exists a real number x such that for all real number y, $x+y=0$.
Solution: For all real numbers x, there is a real number y such that $x+y\neq 0$.

[3] **2**. For each of the following arguments, determine whether the argument is valid or invalid. If it is valid, identify the rule of inference that guarantees it validity. Otherwise, state whether the converse or inverse error is made.
(a) If this number is larger than 2 then its square is larger than 4.
This number is not larger than 2.
$\therefore$ The square of this number is not larger than 4.
Solution: The argument is invalid. The inverse error was made.
(b) If this computer program is correct, then it produces the correct output when run with the test data the teacher gave me.
This computer program produces the correct output when run with the test data the teacher gave me.
$\therefore$This computer program is correct.
Solution: The argument is invalid. The converse error was made.

[9] **3**. Prove or disprove each of the following statements:
(a) For all integers a and b, if $a \mid 10b$ then $a \mid 10$ or $a \mid b$.
Solution: The statement is false because, for example, when $a=30$ and $b=3$, we see that $a \nmid 10$ and $a \nmid b$.
(b) There exists an integer $m \geq 3$ such that m^2-1 is prime.
Solution: The statement is false. Actually, we can prove that for all integers $m \geq 3$, m^2-1 is not prime.

Proof: Let $m \geq 3$ be an integer. Then $m^2-1=(m-1)(m+1)$, so $m-1$ is a factor of m^2-1. We note that since $m \geq 3$, we have that $1<m-1<m^2-1$ and so m^2-1 is not prime (because $m-1$ is a factor of m^2-1).