

MATHEMATICS 271 L01 FALL 2004
ASSIGNMENT 1

Due at 11:00 am on Friday, September 24. Your assignment must be handed in at the beginning of the lecture on September 24. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

Marked assignments will be returned during the lab on Friday, October 3.

1. In this question, prove your answers using **only** the definitions of rational and irrational numbers and the fact that $\sqrt{2}$ is irrational. Let \mathcal{P} be the statement : “For all real numbers x and y , if $x + y$ is rational and $x - y$ is irrational then x is irrational and y is irrational.”

- (a) Is \mathcal{P} true? Prove your answer.
- (b) State the *converse* of \mathcal{P} . Is the *converse* of \mathcal{P} true? Prove your answer.
- (c) State the the *contrapositive* of \mathcal{P} . Is the *contrapositive* of \mathcal{P} true? Explain.
- (d) State the the *inverse* of \mathcal{P} . Is the *inverse* of \mathcal{P} true? Explain.
- (e) State the the *negation* of \mathcal{P} . Is the *negation* of \mathcal{P} true? Explain.

2. In this question, a, b and c are integers. Let \mathcal{Q} be the statement : “For all integers a, b and c , if $a \mid b$ and $a \mid c$ then $a \mid 2b + c$ and $a \mid 3b + 2c$.”.

- (a) Is \mathcal{Q} true? Prove your answer.
- (b) State the *converse* of \mathcal{Q} . Is the *converse* of \mathcal{Q} true? Prove your answer.
- (c) State the the *contrapositive* of \mathcal{Q} . Is the *contrapositive* of \mathcal{Q} true? Explain.
- (d) State the the *inverse* of \mathcal{Q} . Is the *inverse* of \mathcal{Q} true? Explain.
- (e) State the the *negation* of \mathcal{Q} . Is the *negation* of \mathcal{Q} true? Explain.

3. For each of the following statements, determine whether the statement is true or false and **prove your answer**.

- (a) For all integers y , there is an integer x so that $x^3 + x = y$.
- (b) For all integers x and y , if $2x^2 + x = 2y^2 + y$ then $x = y$.
- (c) For all integers m and n , if $m \mid n$ and $n \mid m$ then $n = m$.
- (d) For all rational number x , there is a positive integer k so that kx is an integer.
- (e) There is a positive integer k so that for all rational number x , kx is an integer.