

MATHEMATICS 271 L01 FALL 2007
ASSIGNMENT 1

Due at 12:00 am on Friday, September 28, 2007. Your assignment must be handed in at the beginning of the lab on September 28, 2007. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Let \mathcal{P} be the statement : “For all real numbers x and y , if $x - \lfloor x \rfloor < \frac{1}{2}$ and $y - \lfloor y \rfloor < \frac{1}{2}$ then $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$.”

- (a) Is \mathcal{P} true? Prove your answer.
- (b) State the *converse* of \mathcal{P} . Is the *converse* of \mathcal{P} true? Prove your answer.
- (c) State the *contrapositive* of \mathcal{P} . Is the *contrapositive* of \mathcal{P} true? Explain.
- (d) State the *negation* of \mathcal{P} . Is the *negation* of \mathcal{P} true? Explain.

2. Prove or disprove each of the following statement..

- (a) For all integers a , b and c , if $a \mid b$ and $a \mid c$ then $a \mid xb + yc$ for all integers x and y .
- (b) For all integers a , b and c , $a \mid b$ and $a \mid c$ if and only if $a \mid 2b + c$ and $a \mid b + 2c$.
- (c) For all integers a , b and c , $a \mid b$ and $a \mid c$ if and only if $a \mid 2b + 3c$ and $a \mid b + 2c$.

3. For each of the following statements, determine whether the statement is true or false and prove your answer.

- (a) For all integers x , $x^3 + x$ is even.
- (b) For all integers y , there is an integer x so that $x^3 + x = y$.
- (c) For all integers x and y , if $x^3 + x = y^3 + y$ then $x = y$.