MATHEMATICS 271 L01 FALL 2007
ASSIGNMENT 2

Due at 11:00 a.m. on Monday, October 15. Your assignment must be handed in at the beginning of the lecture on October 15, 2007. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Prove or disprove the following statements:

(a) For real numbers x and y, $\lceil x+y \rceil = \lceil x \rceil + \lceil y \rceil$.

(b) For real numbers x and y, if $x + \lceil x \rceil = y + \lceil y \rceil$ then $x = y$.

(c) For real numbers y, there is a real number x such that $y = x + \lceil x \rceil$.

2. The Fibonacci sequence $f_1,\ f_2,\ f_3 \cdots$ is defined by $f_1 = f_2 = 1$ and for integers $k \geq 3$, $f_k = f_{k-1} + f_{k-2}$.

(a) Prove that $f_n < \left(\frac{7}{4}\right)^{n-1}$ for all integers $n \geq 2$.

(b) Prove that $\sum_{i=1}^{n} {f_i}^2 = f_{n+1} f_n$ for all integers $n \geq 1$.

(c) Prove that $\sum_{i=1}^{n} f_i = f_{n+2} - 1$ for all integers $n \geq 1$.

3. Prove the following statements by induction on n:

(a) $\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > 2\left(\sqrt{n+1} - 1\right)$ for all integers $n \geq 1$.

(b) $\dfrac{1 \cdot 3 \cdot 5 \cdots (2n-3) \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n-2) \cdot (2n)} \geq \dfrac{1}{2n}$ for all integers $n \geq 1$.

(c) $\dfrac{1 \cdot 3 \cdot 5 \cdots (2n-3) \cdot (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n-2) \cdot (2n)} \leq \dfrac{1}{\sqrt{n+1}}$ for all integers $n \geq 1$.