MATHEMATICS 271 L01 FALL 2007
ASSIGNMENT 5

Due at 12:00 noon on Friday, November 30, 2007. Your assignment must be handed in at the beginning of the lab on December 8. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Let $f : A \to B$ be a function and let $\mathcal{S}$ be a relation on B. Let $\mathcal{R}$ be the relation on A defined by "For all $x, y \in A$, $(x, y) \in \mathcal{R}$ if and only if $(f(x), f(y)) \in \mathcal{S}$." Prove or disprove each of the following statement.

(a) If $\mathcal{S}$ is an equivalence relation on B then $\mathcal{R}$ is an equivalence relation on A.

(b) If $\mathcal{R}$ is an equivalence relation on A then $\mathcal{S}$ is an equivalence relation on B.

(c) If $\mathcal{S}$ is antisymmetric then $\mathcal{R}$ is antisymmetric.

(d) If f is one-to-one and $\mathcal{S}$ is antisymmetric then $\mathcal{R}$ is antisymmetric.

2. Let $n \geq 1$ be an integer. Let $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $a \in \mathbb{Z}$. Let $\mathcal{R}$ be the relation on A defined by "For all $x, y \in A$, $(x, y) \in \mathcal{R}$ if and only if $ax \equiv ay \pmod{n}$."

(a) Prove that $\mathcal{R}$ is an equivalence relation on A. Note that you may want to use some result in question 1.

(b) Let $n = 3$ and $a = 2$ Describe the equivalence classes of $\mathcal{R}$.

(c) Let $n = 4$ and $a = 2$. Describe the equivalence classes of $\mathcal{R}$.

(d) Find some integers n and a so that $\mathcal{R}$ has exactly 5 equivalence classes.

(e) Prove that for all integer n, there exists an integer a so that $\mathcal{R}$ has exactly 1 equivalence class.

3. Let $n \geq 1$ be an integer. Let $a, b \in \mathbb{Z}$.

(a) Prove that if $a \equiv c \pmod{n}$ and $b \equiv d \pmod{n}$ then $ab \equiv cd \pmod{n}$.

(b) Prove that if b is an inverse of a modulo n, and $k = b \bmod n$ then k is an inverse of a modulo n.

(c) Use Euclid's Algorithm to find $\gcd(2007, 271)$, and find integers x and y so that $\gcd(2007, 271) = 2007x + 271y$.

(d) Use the result in part (c) to find an inverse of 271 modulo 2007.

(e) Find an integer m so that $1 \leq m < 2007$ so that m is an inverse of 271 modulo 2007.