

MATHEMATICS 271 L01 FALL 2008
ASSIGNMENT 1

This assignment is to be handed in on September 26, 2008 at 12:00 noon. Late assignments will not be accepted and are given a mark of zero. Students should attempt all problems. However, only one problem will be marked for credit.

1. In this question, a , b and c are integers. Prove or disprove each of the following:
 - (a) If $a \mid b$ and $a \mid c$ then $a \mid mb + nc$ for all integers m and n .
 - (b) If $a \mid b$ and $b \mid c$ then $a \mid c$.
 - (c) If $a \mid b - 2c$ and $a \mid 2b + 3c$ then $a \mid b$ and $a \mid c$.
 - (d) If $a \mid b + 2c$ and $a \mid 2b + 3c$ then $a \mid b$ and $a \mid c$.

2. Prove or disprove each of the following:
 - (a) For all real numbers x , there exists a real number y so that $x + y$ is rational.
 - (b) For all real numbers x , there exists a real number y so that $x + y$ and $x + y^2$ are rational.
 - (c) There exists a real number y so that for all real numbers x , $x + y$ is rational.
 - (d) For all real numbers x and y , if xy is irrational then x is irrational or y is irrational.

4. Prove or disprove each of the following:
 - (a) For all integers x , $x^2 - x + 1 > 0$.
 - (b) For all integers x and y , if $2x^2 - x = 2y^2 - y$ then $x = y$.
 - (c) For all integers x and y , if $x^3 + x = y^3 + y$ then $x = y$.
 - (d) For all integers y , there exists an integer x so that $x^3 + x = y$.