MATHEMATICS 271 L01 FALL 2008
ASSIGNMENT 3

Due at noon on Friday, October 24. Your assignment must be handed in at the beginning of the lab on November 10, 2006. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Prove or disprove each of the following statements:

(a) For all sets A and B, $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

(b) For all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

(c) For all sets A, B and C, $(A \cup B) \times (C \cup D) \subseteq (A \times C) \cup (B \times D)$.

(d) For all sets A, B and C, if $A \subseteq (B - C)$ then $A \cap (C - B) = \emptyset$.

2. For any sets A and B, we define the *symmetric difference* $A \triangle B$ by $A \triangle B = (A \cup B) - (A \cap B)$. Note that it is also true that $A \triangle B = (A - B) \cup (B - A)$. Let $\mathcal{S}$ be the statement: "*For all sets A, B and C, if $A \subseteq B \cup C$ and $B \subseteq C \cup A$ then $A \triangle B = C$.*" and let $\mathcal{T}$ be the statement: "*For all sets A, B and C, if $A \triangle B = A \triangle C$ then $B \subseteq C$.*"

(a) Is $\mathcal{S}$ true for all sets A, B and C? Prove your answer.

(b) Is $\mathcal{T}$ true? Prove your answer.

(c) Write the converse of $\mathcal{S}$. Is the converse of $\mathcal{S}$ true? Prove your answer.

3. Let $S = \{1000, 1001, 1002, ..., 9999\}$. For each of the following questions, you must explain how you got the answer.

(a) How many numbers in S have at least one digit that is a 2 or a 5?

(b) How many numbers in S have at least one digit that is a 2 and at least one digit that is a 5?

(c) How many numbers in S have the property that the sum of its digits is even?

(d) How many numbers in S have the property that the digits appear in increasing order (that is, the first digit is smaller than the second digit, the second digit is smaller than the third digit, and the third digit is smaller than the fourth digit)?