

MATHEMATICS 271 L01 FALL 2008
ASSIGNMENT 4

Due at 12:00 noon on Friday, November 14. Your assignment must be handed in at the beginning of the lab on November 14. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Let $h : \mathbb{Z} \rightarrow \mathbb{Z}$ be the function defined by $h(x) = 1 - 2x$ for all $x \in \mathbb{Z}$.

(a) Prove that h is one-to-one but not onto \mathbb{Z} .

(b) Is there a function $g : \mathbb{Z} \rightarrow \mathbb{Z}$ so that $g \circ h$ is onto \mathbb{Z} ?

(c) For positive integers n , define the functions $h^n : \mathbb{Z} \rightarrow \mathbb{Z}$ by putting $h^1 = h$ and for integers $k \geq 2$, $h^k = h \circ h^{k-1}$. Compute $h^2(x)$, $h^3(x)$, $h^4(x)$, and $h^5(x)$. From these, guess an explicit formula for $h^n(x)$ for all integers $n \geq 1$.

(d) Prove by induction on n that your guess in part (c) is correct.

2. Let $f : A \rightarrow A$ and $g : A \rightarrow A$ be functions. Prove or disprove the following statements:

(a) If both f and g are onto then $g \circ f$ is onto.

(b) If $g \circ f$ is onto then f is onto.

(c) If $g \circ f$ is onto then g is onto.

(d) It is possible that $f \circ g = i_A$ but $g \circ f \neq i_A$.

(e) If $f \circ g = i_A$ and f is one-to-one then f is invertible and $f^{-1} = g$.

3. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = 2[x] - x$ for each $x \in \mathbb{R}$.

(a) Prove that f is one-to-one.

(b) Prove that f is onto.

(c) From (a) and (b) we see that f is invertible. Find a formula for $f^{-1}(x)$ for $x \in \mathbb{R}$. Verify that $f \circ f^{-1} = f^{-1} \circ f = i_{\mathbb{R}}$.