MATHEMATICS 271 L01 FALL 2008
ASSIGNMENT 5

Due at 12:00 noon on Monday, December 1, 2008. Assignment must be understandable to the marker (i.e., logically correct as well as legible), and must be done by the student in his / her own words. Answer all questions, but only one question per assignment will be marked for credit. Please make sure that: (i) the cover page has **only** your student ID number, (ii) your name and ID number are on the top right corners of **all** the remaining pages, and (iii) **STAPLE** your papers.

1. Let $\mathcal{R}$ be the relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$ defined by:

"For nay $(a, b), (c, d) \in \mathbb{Z}^+ \times \mathbb{Z}^+$, $(a, b)\,\mathcal{R}\,(c, d)$ if and only if $ab = cd$.

(a) Prove that $\mathcal{R}$ is an equivalence relation on $\mathbb{Z}^+ \times \mathbb{Z}^+$.

(b) List all elements of $[(10, 10)]$ (the equivalence class of $(10, 10)$).

(c) Is there an equivalence class of $\mathcal{R}$ which has exactly 11 elements. Prove your answer.

(d) Is there an equivalence class of $\mathcal{R}$ which has exactly 271 elements. Prove your answer.

2. Let $A = \{1, 2, 3, ..., 10\}$.

(a) How many symmetric relations are there on A?

(b) How many antisymmetric relations are there on A?

(c) How many relations on A are there which are both symmetric and antisymmetric?

(d) How many relations on A are there which are neither symmetric nor antisymmetric?

3. Let $n \geq 1$ be an integer. Let $a, b \in \mathbb{Z}$.

(a) Prove that if $a \equiv c \pmod{n}$ and $b \equiv d \pmod{n}$ then $ab \equiv cd \pmod{n}$.

(b) Prove that if b is an inverse of a modulo n, and $k = b \bmod n$ then k is an inverse of a modulo n.

(c) Use Euclid's Algorithm to find $\gcd(2008, 271)$, and find integers x and y so that $\gcd(2008, 271) = 2008x + 271y$.

(d) Use the result in part (c) to find an inverse of 271 modulo 2008.

(e) Find an integer m so that $1 \leq m < 2008$ so that m is an inverse of 271 modulo 2008.