MATH 271 ASSIGNMENT 1

Due 4:00 PM Friday, October 2, 2009. Hand your assignment to me in class or in the lab, or in my office (MS566, under the door if I'm not there). Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

Note: $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{Q}$ denotes the set of all rational numbers, $\mathbb{Z}$ denotes the set of all integers, and $\mathbb{N}$ denotes the set of all positive integers.

1. For each true statement below, give a proof. For each false statement below, write out its negation, then give a proof of the negation. You may use that every integer is either even or odd (but not both), but otherwise use only the definitions of even and odd integers.

   (a) $\forall a, b \in \mathbb{Z}$, if a is even and $a|b$ then b is even.

   (b) $\forall a, b \in \mathbb{Z}$, if a is even and $b|a$ then b is even.

   (c) $\forall a, b \in \mathbb{Z}$, if a is odd and $a|b$ then b is odd.

   (d) $\forall a, b \in \mathbb{Z}$, if a is odd and $b|a$ then b is odd.

2. Prove or disprove each of the following:

   (a) $\forall a \in \mathbb{N}\ \exists b \in \mathbb{N}$ so that ab is composite.

   (b) $\exists a \in \mathbb{N}$ so that $\forall b \in \mathbb{N}$, ab is composite.

   (c) $\forall a \in \mathbb{N}\ \exists b \in \mathbb{N}$ so that $a + b$ is composite.

   (d) $\exists a \in \mathbb{N}$ so that $\forall b \in \mathbb{N}$, $a + b$ is composite.

3. For this question, do not use Exercises 13–16 on page 146 without proof.

   (a) Prove or disprove: $\forall x \in \mathbb{Q}$, if $x \in \mathbb{Z}$ then $x\lfloor x \rfloor \in \mathbb{Z}$.

   (b) Write out the converse of the statement in (a). Is it true? Give a proof or disproof.

   (c) Prove or disprove: $\forall x \in \mathbb{R}$, if $x \in \mathbb{Q}$ then $x\lfloor x \rfloor \in \mathbb{Q}$.

   (d) Write out the converse of the statement in (c). Is it true? Give a proof or disproof.