MATHEMATICS 271 L20 SPRING 2005
ASSIGNMENT 1

This assignment is to be handed in on June 1, 2005 at 7:00 p.m.. Late assignments will not be accepted and are given a mark of zero. Students should attempt all problems. However, only one problem will be marked for credit.

1. In this question, a, b and c are integers.Prove or disprove each of the following:

(a) If $a \mid b$ and $a \mid c$ then $a \mid mb + nc$ for all integers m and n.

(b) If $a \mid b$ and $b \mid c$ then $a \mid c$.

(c) If $a \mid b - 2c$ and $a \mid 2b + 3c$ then $a \mid b$ and $a \mid c$.

(d) If $a \mid b + 2c$ and $a \mid 2b + 3c$ then $a \mid b$ and $a \mid c$.

2. Prove or disprove each of the following:

(a) For all real numbers x and y, if x is rational and y is irrational then $x + y$ is irrational.

(b) For all real numbers x and y, if x is rational and y is irrational then xy is irrational.

(c) For all real numbers x and y, if $x+y$ is irrational then x is irrational or y is irrational.

(d) For all real numbers x and y, if xy is irrational then x is irrational or y is irrational.

3. Let $\mathcal{P}$ be the statement: "for all real numbers x, if $x < \lfloor x \rfloor + \frac{1}{2}$ then $\lfloor 2x \rfloor = 2 \lfloor x \rfloor$." and let Q be the statement: "for all positive real numbers x and y, if $\lfloor x \rfloor = \lfloor y \rfloor$ then $\lfloor x^2 \rfloor = \lfloor y^2 \rfloor$."

(a) Is $\mathcal{P}$ true? Prove your answer.

(b) Write the converse of $\mathcal{P}$. Is the converse $\mathcal{P}$ of true? Prove your answer.

(c) Write the contrapositive of $\mathcal{P}$. Is the contrapositive $\mathcal{P}$ of true? Prove your answer.

(d) Is $\mathcal{Q}$ true? Prove your answer.

(e) Write the converse of $\mathcal{Q}$. Is the converse $\mathcal{Q}$ of true? Prove your answer.

4. Prove or disprove each of the following:

(a) For all integers x, $x^2 - x + 1 > 0$.

(b) For all integers x and y, if $2x^2 - x = 2y^2 - y$ then $x = y$.

(c) For all integers x and y, if $x^3 + x = y^3 + y$ then $x = y$.

(d) For all integers y, there exists an integer x so that $x^3 + x = y$.