MATHEMATICS 271 L20 SPRING 2005
ASSIGNMENT 3

This assignment is to be handed in on Wednesday, June 22, 2005 at 7:00 p.m.. Late assignments will not be accepted and are given a mark of zero. Students should attempt all problems. However, only one problem will be marked for credit.

1.Let $f : \mathbb{R} \longrightarrow \mathbb{R}$ be a function defined by $f(x) = x + \lceil x \rceil$ for all $x \in \mathbb{R}$.

(a) Is f one-to-one? Prove your answer.

(b) Is f onto? Prove your answer.

(c) Describe the largest subset B of $\mathbb{R}$ for which the function $g : \mathbb{R} \longrightarrow B$ defined by $g(x) = x + \lceil x \rceil$ for all $x \in \mathbb{R}$ is an one-to-one and onto function, and find an explicit formula for $g^{-1}(x)$ for each $x \in B$.

2. Let $A = \{1, 2, 3, ..., n\}$ where $n \geq 2$.

(a) Find the number of functions $f : A \to A$ such that $f \circ f(1) = 2$.

(b) Find the number of functions $f : A \to A$ such that f is onto.

(c) Find the number of functions $f : A \to A$ such that f is onto and $f \circ f(1) = 2$.

(d) Find the number of functions $f : A \to A$ such that f is onto or $f \circ f(1) = 2$.

3. Let $f : A \to B$ and $g : B \to C$ be functions. Prove or disprove each of the following.

(a) If f and g are one-to-one then $g \circ f$ is also one-to-one.

(b) If $g \circ f$ is one-to-one then f must be one-to-one.

(c) If $g \circ f$ is one-to-one then g must be one-to-one.

(d) If $g \circ f$ is one-to-one and f is onto then g is one-to-one.

4. Let $A = \{1, 2, 3, 4\}$. Let $\mathcal{R}$ and $\mathcal{S}$ be relations on A. Prove or disprove each of the following statements:

(a) If $\mathcal{R}$ and $\mathcal{S}$ are symmetric then $\mathcal{R} \cup \mathcal{S}$ is symmetric.

(b) If $\mathcal{R} \cup \mathcal{S}$ is symmetric then $\mathcal{R}$ and $\mathcal{S}$ are symmetric.

(c) There exists a relation $\mathcal{T}$ on A so that $\mathcal{T}$ is not reflexive but $\mathcal{I}$ is both symmetric and antisymmetric.

(d) There exists a relation $\mathcal{V}$ on A so that $\mathcal{V}$ is not transitive but $\mathcal{V}$ is both symmetric and antisymmetric..