

## Sample Final for Math 271

August 10, 2005

NAME:.....

**Instructions.** You have 3 hours to complete this test. Please provide detailed solutions for the exercises. Only complete answers with sufficient explanation are worth full credit. No open textbooks or notes are allowed.

1. Write the converse and the inverse of the following statements.
  - a) (5 marks) If Tom is Ann's father, then Jim is her uncle and Sue is her aunt.
  - b) (5 marks) If today is New Year's Eve, then tomorrow is January.
2. (10 marks) True or false?  $\forall$  positive integers  $a, b$  and  $c$ , if  $a \mid bc$  then  $a \mid b$  or  $a \mid c$ ? Explain.
3. a) (5 marks) Determine whether the following function is one-to-one.

$$f(x) = \frac{x^2}{x^2 - x - 1}$$

- b) (5 marks) Let  $f : \mathbf{R} \rightarrow \mathbf{R}$  and  $f^{-1} : \mathbf{R} \rightarrow \mathbf{R}$  defined by

$$f(x) = 3x + 2$$

and

$$f^{-1}(x) = \frac{y - 2}{3}.$$

Determine whether the functions are inverse to each other.

4. a) (5 marks) Give two examples of graphs that have Euler circuits and Hamiltonian circuits that are not the same.
  - b) (5 marks) Draw a tree with six vertices and total degree 18 or explain why such graph does not exist.
5. A computer programming team has 13 members. Suppose that seven members are women and six are men.
  - a) (5 marks) How many groups of seven can be chosen that contain at least one man?
  - b) (5 marks) What is the probability that a group of seven contains at most three women?
6. (10 marks) Find the coefficient of  $p^{16}q^7$  in  $(3p^2 - 2q)^{15}$ .

7. (10 marks) Suppose that  $A$  and  $B$  are events in a sample space  $S$  with probabilities 0.8 and 0.7 respectively. Suppose also that  $P(A \cap B) = 0.6$ . What is  $P(A \cup B)$ ?
8. (10 marks) Prove by induction.

$$\prod_{i=0}^n \left( \frac{1}{2i+1} \cdot \frac{1}{2i+2} \right) = \frac{1}{(2n+2)!}.$$

9. (10 marks) Prove by induction.

$$\sum_{i=1}^n i(i!) = n \cdot 2^{n+2} + 2.$$

10. (10 marks) Given sets  $A$  and  $B$ , the *symmetric difference* of  $A$  and  $B$ , denoted  $A \triangle B$ , is  $A \triangle B = (A - B) \cup (B - A)$ . Prove that if  $A \triangle C = B \triangle C$ , then  $A = B$ .