Quiz 4 for Math 271

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Instructions. You have 35 minutes to complete this quiz. Please provide detailed solutions for the exercises. Only complete answers with sufficient explanation are worth full credit. No open textbooks or notes are allowed.

1. (7 marks) Let A be a set with at least two elements and $\mathcal{P}(A)$ the power set of A. Define a relation $\mathcal{R}$ on $\mathcal{P}(A)$ as follows. For all $X, Y \in \mathcal{P}(A)$ $\quad X \mathcal{R} Y \Leftrightarrow X \subseteq Y$ or $Y \subseteq X$. Determine whether $\mathcal{R}$ is reflexive, symmetric, transitive, or none of these.

reflexivity: $X \in \mathcal{P}(A) \Rightarrow X \subseteq X \Rightarrow X R X$ ✓

symmetry: $X, Y \in \mathcal{P}(A)$ and $X R Y \Rightarrow X \subseteq Y$ or $Y \subseteq X \Rightarrow$
$\Rightarrow Y R X$ ✓

transitivity: not transitive. counterexample: Let $A = \{1, 2, 3\}$,
$X = \{1, 2\}$, $Y = \{1, 2, 3\}$, $Z = \{2, 3\}$. Then $X \subseteq Y$,
$Z \subseteq Y$ but $X \not\subseteq Z$ and $Z \not\subseteq X$.

2. (7 marks) Define a relation R on the set $\mathbf{Z}$ of all integers as follows:
$m \, R \, n \Leftrightarrow$ every prime factor of m is a prime factor of n
Determine whether R is a partial order relation.

reflexivity: $m \in \mathbb{Z} \Rightarrow m R m$ ✓

antisymmetry: $m, n \in \mathbb{Z}$, let $m = 3$ and $n = 9$.
They have the same prime factors but
$3 \neq 9$. Therefore R is not antisymmetric.
It is Not a partial order relation.

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3. (6 marks) Two faces of a six-sided die are painted red, two are painted blue, and two are painted yellow. The die is rolled three times, and the colors that appear face up on the first, second and third rolls are recorded.

a) Consider the event that all three rolls produce different colors. What is the probability of the event?

b) Consider the event that two of the colors that appear face up are the same. What is the probability of the events?

Notice that each colour has the same likelyhood.
The number of all possible outcomes of the experiment is $3 \cdot 3 \cdot 3 = 27$. $\underset{1^{st}}{\overset{3}{\square}}\ \underset{2^{nd}}{\overset{3}{\square}}\ \underset{3^{rd}}{\overset{3}{\square}}$

a) When all 3 colours are diffent the we get a permutation of the letters B, R, Y.
There are $3! = 6$ such permutations.

Marks:

1)............. Therefore $p = \frac{6}{27} = \frac{2}{9}$

2).............

3).............

total:..............

b) There are 3 ways we can get all three colours the same: BBB, RRR, YYY.
Thus, out of the 27 possible outcomes 3 have all 3 colours the same, and 6 have three different colours. Therefore the number of possible outcomes where exactly 2 colours are the same is $27 - 6 - 3 = 18$. $p = \frac{18}{27} = \frac{2}{3}$.
The number of possible outcomes where at least 2 colours are the same is $27 - 6 = 21$. $p = \frac{21}{27} = \frac{7}{9}$.