

Quiz 2 for Math 271

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**Instructions.** You have 35 minutes to complete this quiz. Please provide detailed solutions for the exercises. Only complete answers with sufficient explanation are worth full credit. No open textbooks or notes are allowed.

1. (5 marks) True or false? For all integers  $n$ ,  $n^2 - n + 1$  is a prime number. Explain.

The statement is FALSE. Here is a counterexample.  
Let  $n=0$ . Then  $0^2 - 0 + 1 = 1$  which is not a prime.

2. (5 marks) Prove or disprove: For all integers  $a$  and  $n$ , if  $a \mid n^2$  and  $a \leq n$  then  $a \mid n$ .

The statement is FALSE. Here is a counterexample.

Let  $a=4$  and  $n=10$ .

$a \mid n^2$  since  $4 \mid 100$  but  $4 \nmid 10$ .

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3. (5 marks) Prove or disprove: The sum of any two positive irrational numbers is irrational.

The statement is FALSE. Here is a counterexample.

Let  $r = \sqrt{2}$  and  $s = 2 - \sqrt{2}$ . Both  $r$  and  $s$  are irrational. However,  $r + s = \sqrt{2} + (2 - \sqrt{2}) = 2$  which is rational.

4. (5 marks) Prove the statement. If  $a, b$  are rational numbers,  $b \neq 0$ , and  $r$  is an irrational number, then  $a + br$  is irrational.

Proof: (by contradiction) Assume, on the contrary, that  $a, b \in \mathbb{Q}$ ,  $b \neq 0$ ,  $r$  irrational, and  $a + br$  is rational. Then  $a = \frac{x}{y}$ ,  $b = \frac{v}{w}$ ,  $a + br = \frac{p}{s}$  for some integers  $x, y, v, w, p, s$ .

Marks:

1).....

2).....

3).....

4).....

total:.....

$$a + br = \frac{x}{y} + r \frac{v}{w} = \frac{p}{s}$$

$$r = \frac{\frac{p}{s} - \frac{x}{y}}{\frac{v}{w}} = \frac{\frac{py - xs}{sy}}{\frac{v}{w}} = \frac{w(py - xs)}{syv}$$

Both  $w(py - xs)$  and  $syv$  are integers, so  $r$  is rational. This is a contradiction.  $\square$