Quiz 5 for Math 271

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Instructions. You have 35 minutes to complete this quiz. Please provide detailed solutions for the exercises. Only complete answers with sufficient explanation are worth full credit. No open textbooks or notes are allowed.

1. (7 marks) A bakery produces six kinds of pastry.

   a) How many selections of 10 pastries are there?

   b) If a selection of twenty pastries are chosen randomly, what is the probability that at least three are eclairs?

a, Since one kind of pastry can be chosen more than once, we use combinations with repetitions. $n = 6$, $r = 10$ $\binom{6+10-1}{10} = \binom{15}{10}$.

b, Choose 3 eclairs first, the choose the other 17 pastries from the six possible kinds. $\binom{6+17-1}{17} = \binom{22}{17}$

The total number of choices of 20 pastries: $\binom{6+20-1}{20} = \binom{25}{20}$

Therefore, the probability of the even: $\dfrac{\binom{22}{17}}{\binom{25}{20}}$.

2. (7 marks) Prove by mathematical induction that if n is an integer and $n \geq 1$, then

$$\sum_{i=2}^{n+1} \binom{i}{2} = \binom{2}{2} + \binom{3}{2} + \cdots + \binom{n+1}{2} = \binom{n+2}{3}$$

Step 1: $n=1$ $\sum_{i=2}^{2} \binom{i}{2} = \binom{2}{2} = \binom{3}{3} = 1$ ✓

Step 2: Let $k \geq 1$ Assume that $\sum_{i=2}^{k+1} \binom{i}{2} = \binom{k+2}{3}$

We must show that $\sum_{i=2}^{k+2} \binom{i}{2} = \binom{k+3}{3}$.

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$LHS = \sum_{i=2}^{k+1} \binom{i}{2} + \binom{k+2}{2} = \binom{k+2}{3} + \binom{k+2}{2} = \binom{k+3}{3} = RHS$ ✓

by Pascal's Formula

3. (6 marks) How many onto functions are there from a set of four elements to a set with three elements?

Let $N(X) = 4$ and $N(Y) = 3$.

Total number of functions $f: X \to Y = 3^4 = 81$

If $f: X \to Y$ is not onto, then the image $f(X)$ contains only 2 elements (at most) of Y. There are $\binom{3}{2} = 3$ choices for these 2 elements. The number of fuctions whose range is a fixed set of two elements is 2^4.

However, this way we count the functions, whose range consists of only one element, twice. Therefore, the number of functions $f: X \to Y$ that are not onto is $\binom{3}{2} 2^4 - 3 = 45$

The number of onto functions: $81 - 45 = \underline{36}$.

Marks:

1).............

2).............

3).............

total:.............