University of Calgary
Department of Mathematics and Statistics

MATH 271 (L60)

Date of exam	**QUIZ 2**	**Duration of exam**
July 17, 2007		**35 minutes**

STUDENT'S ID: SOLUTION KEY

INSTRUCTIONS: No calculators, open book or formula sheets.

1. [4 marks] True or false? If k is any even integer and m is any odd integer, then $(k+2)^2-(m-1)^2$ is even. Explain.

TRUE Let k be an even, m is an odd integer

- k even $\Rightarrow$ $k+2$ even $\Rightarrow$ $(k+2)^2 = (k+2)(k+2)$ even
- m odd $\Rightarrow$ $m-1$ even $\Rightarrow$ $(m-1)^2 = (m-1)(m-1)$ even

$(k+2)^2 - (m-1)^2 =$ even $-$ even $=$ even. $\square$

2. [5 marks] Determine whether the following statement is true or false. If it is true then give a proof, if it is false then provide a counterexample.

For all integers a, b and c, if $a \mid bc$ then $a \mid b$ or $a \mid c$.

False.

Let $a=6$, $b=3$, $c=4$.

$a \mid bc$: $6 \mid 3 \cdot 4 = 12$ but

$6 \nmid 3$ and $6 \nmid 4$.

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3. [5 marks] Prove the following statement.
The fourth power of any integer has the form $8m$ or $8m+1$ for some integer m.

Let n be an arbitrary integer.

Case 1: "n even" $\Rightarrow \exists k \in \mathbb{Z}$, $n = 2k$. $\Rightarrow n^4 = (2k)^4 =$
$= 2^4 k^4 = 16k^4 = 8(2k^4) = 8m$, where $m = 2k^4$.

Case 2: "n odd" $\Rightarrow \exists l \in \mathbb{Z}$, $n = 2l+1 \Rightarrow n^4 = (2l+1)^4 =$
$= (2l+1)^2(2l+1)^2 = (4l^2+4l+1)(4l^2+4l+1) =$
$= 16l^4 + 16l^3 + 4l^2 + 16l^3 + 16l^2 + 4l + 4l^2 + 4l + 1 =$
$= 16l^4 + 32l^3 + 24l^2 + 8l + 1 = 8\underbrace{(2l^4 + 4l^3 + 3l^2 + l)}_{m} + 1 = 8m+1$

$\square$

4 [6 marks] Prove the following statement.
For any integer m and any real number x, if x is not an integer, then $\lfloor x \rfloor + \lfloor m - x \rfloor = m - 1$.

Let $x \in \mathbb{R}$ and $x \notin \mathbb{Z}$ and $m \in \mathbb{Z}$. Then
$n < x < n+1$ for some integer n. (Of course, in this case $n = \lfloor x \rfloor$.)
It follows that $-n > -x > -n-1$, and $-n+m > -x+m > -n-1+m$
Thus, $m-n-1 < m-x < m-n$ and by definition
$\lfloor m-x \rfloor = m-n-1$. So $\lfloor x \rfloor + \lfloor m-x \rfloor = n + (m-n-1) = \underline{\underline{m-1}}$

$\square$

MARKS:

1).........................

2).........................

3).........................

4).........................

Total:.....................