University of Calgary
Department of Mathematics and Statistics

MATH 271 (L60)

Date of exam	QUIZ 3	Duration of exam
July 31, 2007		35 minutes

STUDENT'S ID: SOLUTION KEY

INSTRUCTIONS: No calculators, open book or formula sheets.

1. [5 marks] Prove the following statement using an element argument.

For all sets A, B and C, if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$.

Let A, B and C sets such that $A \subseteq B$ and $A \subseteq C$.
Let $x \in A$. $A \subseteq B \Rightarrow x \in B$,
$A \subseteq C \Rightarrow x \in C$
Therefore $x \in B \cap C$. Thus for all $x \in A$, $x \in B \cap C$
which is equivalent to $A \subseteq B \cap C$.

2. [5 marks] Prove the following statement using an element argument.

For all sets A, B and C, if $C \subseteq B - A$ then $A \cap C = \emptyset$.

Let A, B and C any sets such that $C \subseteq B - A$. $\exists\, x \in A \cap C$.
On the contrary, assume that $A \cap C \neq \emptyset$. $\Rightarrow$
$\Rightarrow x \in A$ and $x \in C$. Since $C \subseteq B - X$, $x \in B - A$. $\Rightarrow$
$\Rightarrow x \in B$ and $x \notin A$. However, $x \in A$ and $x \notin A$ at the same
time is a contradiction. Thus the supposition on the
contrary is false. $\square$

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3. [5 marks] Prove the statement if it is true and give a counterexample if it is false.

For all sets A, B and C, $A - (B - C) = (A - B) - C$.

False.

Counterexample.

$A = B = C = \{1\}$

$LHS = A - (B - C) = \{1\} - \emptyset = \{1\}$

$RHS = (A - B) - C = \emptyset - \{1\} = \emptyset$

$LHS \neq RHS$

4 [5 marks] Provide an algebraic proof of the following statement.

For all sets A and B, $((A^c \cup B^c) - A)^c = A$.

$((A^c \cup B^c) - A)^c = ((A^c \cup B^c) \cap A^c)^c = (A^c \cup B^c)^c \cup A =$

$= (A \cap B) \cup A = A \cup (A \cap B) = A$ □

MARKS:

1).....

2).....

3).....

4).....

Total:.....