University of Calgary
Department of Mathematics and Statistics

MATH 271 (L60)

Date of exam	QUIZ 5	Duration of exam
August 14, 2007		35 minutes

STUDENT'S ID: SOLUTION KEY

INSTRUCTIONS: No calculators, open book or formula sheets.

1. [5 marks] Prove that if S is any sample space and U and V are any events in S, then $P(V-U) = P(V) - P(U \cap V)$.

Notice that $V = (V-U) \cup (U \cap V)$ and

$(V-U) \cap (U \cap V) = \emptyset$.

Therefore

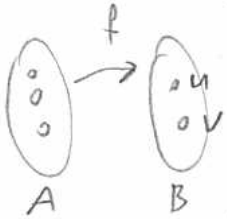
$$P(V-U) + P(U \cap V) = P(V)$$

$$P(V-U) = P(V) - P(U \cap V)$$

2. [4 marks] a) How many onto functions are there from a set of three elements to a set of two elements?

b) How many onto functions are there from a set of three elements to a set of five elemenets?

a)



Two elements of A must be mapped into the same element of B. There are $\binom{3}{2}$ ways a 2-element subset can be chosen from a set of 3 elements. There are two choices where the 2-element subset is mapped: u or v. Thus, altogether there are $2 \cdot \binom{3}{2}$ such functions. $2 \cdot \binom{3}{2} = 2 \cdot 3 = 6$

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3. [5 marks] If $f : X \to Y$ and $g : Y \to Z$ are functions and $g \circ f$ is onto, must g be onto? Prove or give a counterexample.

Clearly, the answer is YES. Assume, on the contrary, that g is not onto. Then $\exists\, b \in B$ which is not in the range of g, and which cannot be in the range of $g \circ f$ either. This contradicts to the assumption that $g \circ f$ is onto.

4 [6 marks] A is the "absolute value" relation on $\mathbf{R}$: For all real numbers x and y, $xAy \iff |\,x\,| = |\,y\,|$. Determine whether the relation is reflexive, symmetric, transitive, or none of these. Justify your answers.

reflexivity: $x \in \mathbb{R} \Rightarrow |x| = |x| \Rightarrow xAx$ ✓

symmetry: $x, y \in \mathbb{R}$ and $xAy \Rightarrow |x| = |y| \Rightarrow |y| = |x| \Rightarrow yAx$ ✓

transitivity: $x, y, z \in \mathbb{R}$ and $xRy,\ yRz \Rightarrow |x| = y$ and $|y| = z$

$\Rightarrow |x| = |z| \Rightarrow xAz$ ✓

MARKS:

1).....

2).....

3).....

4).....

Total:.....