MATH 271 ASSIGNMENT 2

Due 4:00 PM Friday, February 11, 2005. Put your assignment in the appropriate **wooden slot** (corresponding to your lecture section and last name) inside room MS 315. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. (a) Find all positive integers a so that $\lfloor a/271 \rfloor = 10$. How many such integers are there?
   (b) Find all positive integers a so that $\lfloor 271/a \rfloor = 10$.
   (c) Find all positive integers a so that $\lceil 271/a \rceil = 10$.
   (d) Prove or disprove: $\forall n \in \mathbf{Z}$, the equations $\lfloor 271/x \rfloor = n$ and $\lceil 271/x \rceil = n$ have the *same number* of integer solutions x.
   (e) Prove or disprove: $\exists n \in \mathbf{Z}$ so that $\forall a \in \mathbf{Z}$, $\lfloor 271/a \rfloor \neq n$.

2. Let
$$S_n = \frac{3}{1 \cdot 2} - \frac{5}{2 \cdot 3} + \frac{7}{3 \cdot 4} - \frac{9}{4 \cdot 5} + \cdots - \frac{4n-3}{(2n-2)(2n-1)} + \frac{4n-1}{(2n-1)2n},$$
   where the signs alternate.
   (a) Calculate and simplify S_1, S_2 and S_3.
   (b) Use part (a) (and more calculations if you need them) to guess a simple formula for S_n.
   (c) Prove your formula for all positive integers n *using mathematical induction.*
   (d) Give another proof of your formula for all positive integers n *using telescoping.* (See example 4.1.10 on page 205 of the text.)

3. (a) Prove the following statement by contradiction: for all integers n, if $3|n$ then $3 \nmid (n+271)$.
   (b) Prove or disprove: for all integers n, if $3|n$ then $5 \nmid n$.
   (c) Prove by mathematical induction that $3|(2^n - (-1)^n)$ for all integers $n \geq 1$.