MATH 271   ASSIGNMENT 1

Due 4:00 PM Friday, January 28, 2005. Put your assignment in the appropriate **wooden slot** (corresponding to your lecture section and last name) inside room MS 315. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. (a) Let $\mathcal{S}$ be the statement

   For all integers n, if n is even then $3n - 11$ is odd.

   Is $\mathcal{S}$ true? Give a proof or counterexample.

   (b) Write out the *contrapositive* of statement $\mathcal{S}$, and give a proof or disproof.

   (c) Write out the *converse* of statement $\mathcal{S}$, and give a proof or disproof.

   (d) Prove or disprove the statement

   For all integers n, if n is odd then $2n - 11$ is even.

   Then write out the converse of this statement and prove or disprove it.

2. Prove or disprove the following statements:

   (a) There exists a prime number a such that $a + 271$ is prime.

   (b) There exists a prime number a such that $a + 271$ is composite.

   (c) There exists a composite number a such that $a + 271$ is prime.

   (d) There exists a composite number a such that $a + 271$ is composite.

   (e) Choose one of statements (a) to (d) (your choice), replace 271 with your U of C ID number, and prove or disprove the resulting statement.

3. *Note*: $\mathbf{Z}$ denotes the set of all integers, and $\mathbf{Z}^+$ denotes the set of all positive integers.

   (a) Prove the following statements:

   (i) $\exists a \in \mathbf{Z}$ so that $\forall b \in \mathbf{Z}, (a-b) | (a+b)$.

   (ii) $\forall a \in \mathbf{Z}^+ \; \exists b \in \mathbf{Z}^+$ so that $(a-b)|(a+b)$.

   (iii) $\forall a \in \mathbf{Z}^+ \; \exists b \in \mathbf{Z}^+$ so that $(a+b)|(a-b)$.

   (b) Write out the *negation* of the following statement:

   $\forall a, b \in \mathbf{Z}^+$, if $a|2$ and $b|3$ then $(a+b)|5$.

   Then show that the negation is true, so that the original statement is false.

   (c) Prove the following statement:

   $\exists N \in \mathbf{Z}^+$ so that $\forall a, b \in \mathbf{Z}^+$, if $a|2$ and $b|3$ then $(a+b)|N$.