MATH 271 ASSIGNMENT 3

Due 4:00 PM Friday, March 4, 2005. Put your assignment in the appropriate **wooden slot** (corresponding to your lecture section and last name) inside room MS 315. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. (a) Prove **by induction** that, for all integers $n \geq 2$,
$$\frac{1^2}{2!} + \frac{2^2}{3!} + \frac{3^2}{4!} + \cdots + \frac{n^2}{(n+1)!} \leq 2 - \frac{2n}{(n+1)!} \ . \tag{1}$$
   (b) Prove that in fact inequality (1) holds for all integers $n \geq 1$.
   (c) Find the smallest real number A so that, for all integers $n \geq 1$,
$$\frac{1^2}{2!} + \frac{2^2}{3!} + \frac{3^2}{4!} + \cdots + \frac{n^2}{(n+1)!} \leq A - \frac{2n}{(n+1)!} \ .$$

2. You are given the following "while" loop:

   [*Pre-condition*: m is a nonnegative integer, $a = 0$, $b = 1$, $c = 2$, $i = 0$.]

   while $(i \neq m)$

   1. $a := b$
   2. $b := c$
   3. $c := 2b - a$
   4. $i := i + 1$

   end while

   [*Post-condition*: $c = m + 2$.]

   Loop invariant: $I(n)$ is "$a = n$, $b = n + 1$, $c = n + 2$, $i = n$".

   (a) Prove the correctness of this loop with respect to the pre- and post-conditions.
   (b) Suppose the "while" loop is as above, except that the pre-condition is replaced by: m is a nonnegative integer, $a = 1$, $b = 3$, $c = 5$, $i = 0$. Find a post-condition that gives the final value of c, and an appropriate loop invariant, and prove the correctness of this loop.

3. Prove or disprove each of the following six statements. Proofs should use the "element" methods given in Section 5.2. [*Note*: $\mathcal{P}(X)$ denotes the power set of the set X.]

   (a) For all sets A, B, C, $(A - B) \times C \subseteq (A \times C) - (B \times C)$.
   (b) For all sets A, B, C, $(A \times C) - (B \times C) \subseteq (A - B) \times C$.
   (c) For all sets A, B, C, $(A - B) \times C = (A \times C) - (B \times C)$.
   (d) For all sets A and B, $\mathcal{P}(A - B) \subseteq \mathcal{P}(A) - \mathcal{P}(B)$.
   (e) For all sets A and B, $\mathcal{P}(A) - \mathcal{P}(B) \subseteq \mathcal{P}(A - B)$.
   (f) For all sets A and B, $\mathcal{P}(A - B) = \mathcal{P}(A) - \mathcal{P}(B)$.