MATH 271 ASSIGNMENT 1

Due 4:00 PM Friday, February 2, 2007. Put your assignment in the appropriate **wooden slot** (corresponding to your lecture section and last name) inside room MS 315. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

Note: $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{R}^+$ denotes the set of all positive real numbers, $\mathbb{Z}$ denotes the set of all integers, and $\mathbb{Z}^+$ denotes the set of all positive integers.

1. For each true statement below, give a proof. For each false statement below, write out its negation, then give a proof of the negation.
   (a) $\forall a, b \in \mathbb{Z}^+$, if $a|b$ and $(a+1)|b$ then $(a+2)|b$.
   (b) $\forall a, b \in \mathbb{Z}^+$, if $a|b$ and $a|(b+1)$ then $a|(b+2)$.
   (c) $\exists a \in \mathbb{Z}^+$ such that $\forall b \in \mathbb{Z}^+$, $a|b$ and $(a+1)|b$.
   (d) $\forall a \in \mathbb{Z}^+$ $\exists b \in \mathbb{Z}^+$ such that $a|b$ and $(a+1)|b$.
   (e) $\forall a \in \mathbb{Z}^+$ $\exists b \in \mathbb{Z}^+$ such that $a < b$, $a|b$ and $(a+1)|(b+1)$.
2. (a) Prove or disprove the following statement: $\forall a \in \mathbb{R}$, if $\lfloor a \rfloor = 2$ then $\lfloor 2a \rfloor = 4$.
   (b) Write out the contrapositive of the statement in part (a). Is it true or false? Explain.
   (c) Write out the converse of the statement in part (a). Is it true or false? Explain.
   (d) Prove or disprove the following statement: $\forall r \in \mathbb{R}^+$ $\exists n \in \mathbb{Z}^+$ so that $\lfloor rn \rfloor$ is prime.
   (e) Prove or disprove the following statement: $\exists n \in \mathbb{Z}^+$ so that $\forall r \in \mathbb{R}^+$, $\lfloor rn \rfloor$ is prime.
3. Let N be your student ID number.
   (a) **Use the Euclidean Algorithm** to find $\gcd(N, 271)$.
   (b) Use your answer to part (a) to write $\gcd(N, 271)$ in the form $Na + 271b$ where $a, b \in \mathbb{Z}$.
   (c) [In this part you may use results from §3.1 such as Theorem 3.1.1 on page 133 or exercises 25, 26, 27, 39, 40 or 42 from page 140. If you use any of these results, be sure to say which ones.]
   Let's consider all the Math 271 students' answers to part (b). Prove that no student could have correctly given integers a and b which are both even.