MATH 271 ASSIGNMENT 2

Due 4:00 PM Friday, February 16, 2007. Put your assignment (stapled, please) in the appropriate **wooden slot** (corresponding to your lecture section and last name) inside room MS 315. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. For an integer $n \geq 1$, let $S(n)$ be the statement
$$2 + \frac{1}{24} - \frac{2}{n+1} \leq \frac{1}{1^3} + \frac{3}{2^3} + \frac{5}{3^3} + \cdots + \frac{2n-1}{n^3} \leq 3 - \frac{2}{n} .$$
   (a) Prove **by induction** (or by well-ordering) that $S(n)$ is true for all integers $n \geq 2$.
   (b) Let N be your student ID number. Use (a) to find
$$\left\lfloor \frac{1}{1^3} + \frac{3}{2^3} + \frac{5}{3^3} + \cdots + \frac{2N-1}{N^3} \right\rfloor .$$

2. The sequence $b_0, b_1, b_2, \ldots$ is defined by: $b_0 = 1$, $b_1 = 1$, $b_2 = 6$, and $b_n = 3b_{n-2} + 2b_{n-3}$ for all integers $n \geq 3$.
   (a) Find b_3, b_4 and b_5.
   (b) Use part (a) (and more data if you need it) to guess a simple formula for b_n.
   [*Hint*: how far away is b_4 from the *nearest* power of 2? How about b_5?]
   (c) Use **strong induction** (or well-ordering) to prove your guess.

3. You are given the following "while" loop:

   [*Pre-condition*: m is a nonnegative *even* integer, $a = 0$, $b = 0$, $c = 0$.]

   while $(a \neq m)$
   1. $b := 2a - b$
   2. $c := 2b - c$
   3. $a := a + 1$

   end while

   [*Post-condition*: $c = 2m$.]

   Loop invariant: $I(n)$ is
$$a = n, \quad b = \left\{ \begin{array}{ll} n & \text{if } n \text{ is even} \\ n-1 & \text{if } n \text{ is odd} \end{array} \right\}, \quad c = \left\{ \begin{array}{ll} 2n & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{array} \right\}.$$
   (a) Prove the correctness of this loop with respect to the pre- and post-conditions.
   (b) Suppose the "while" loop is as above, but $c = 1$ in the pre-condition, and statement 2 in the "while" loop is replaced by: $c := 2b - a$. Find a post-condition that gives the final value of c, and an appropriate loop invariant, and prove the correctness of this loop.