MATH 271 ASSIGNMENT 3

Due 4:00 PM Friday, March 16, 2007. Put your assignment (stapled, please) in the appropriate **wooden slot** (corresponding to your lecture section and last name) inside room MS 315. Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

1. Let n be a positive integer. If $A_1, A_2, \ldots, A_n$ are sets, we write

$$\mathcal{S}_n = A_1 - (A_2 - (A_3 - (\cdots - (A_{n-1} - A_n))\cdots)).$$

For example, if $n = 4$ then $\mathcal{S}_4 = A_1 - (A_2 - (A_3 - A_4))$.

   (a) Let A be a set, and let $A_i = A$ for all $1 \le i \le n$, so that

$$\mathcal{S}_n = A - (A - (A - (\cdots - (A - A))\cdots)),$$

   where there are n A's. Prove (using induction or well ordering) that

$$\mathcal{S}_n = \begin{cases} A & \text{if } n \text{ is odd} \\ \emptyset & \text{if } n \text{ is even.} \end{cases}$$

   (b) Prove that for all sets A and B, $A - (B - A) = A$. You may use the identities on page 272.

   (c) Let A and B be sets, and let $A_i = \begin{cases} A & \text{if } i \text{ is odd} \\ B & \text{if } i \text{ is even} \end{cases}$. Find a simple formula (something like in part (a)) for $\mathcal{S}_n$, and prove it using induction or well ordering.

2. There are 5 men and 5 women, of 10 different heights.

   (a) Find the number of ways of arranging the 10 people in a row so that the ith shortest woman is next to the ith shortest man, for all $1 \le i \le 5$.

   (b) Find the number of ways of arranging the 10 people in a row so that the women occupy five consecutive spots.

   (c) Find the number of ways of arranging the 10 people in a row so that everyone except the tallest person is next to someone taller.

3. Find the number of ordered pairs (A, B) of subsets of $\{1, 2, \ldots, 10\}$ satisfying:

   (a) $N(A \cap B) = 7$. ($N(X)$ is the number of elements in the set X; see page 299.)

   (b) $N(A \times B) = 7$.

   (c) $N\big(\mathcal{P}(A \cup B)\big) = 7$. ($\mathcal{P}(X)$ is the power set of the set X.)

   (d) $N\big(\mathcal{P}(A) \cup \mathcal{P}(B)\big) = 7$.