MATH 271 ASSIGNMENT 1

Due 4:00 PM Friday, February 8, 2008. Hand your assignment to the marker (Jason Nicholson) in his office MS 390. (Or if nobody is in his office, put your assignment under the door.) Assignments must be understandable to the marker (i.e., logically correct as well as legible), and of course must be done by the student in her/his own words. Answer **all** questions; but only one question per assignment will be marked for credit.

Marked assignments will be handed back during your scheduled lab, or in class.

Note: $\mathbb{R}$ denotes the set of all real numbers, $\mathbb{Q}$ denotes the set of all rational numbers, and $\mathbb{Z}$ denotes the set of all integers.

1. For each true statement below, give a proof. For each false statement below, write out its negation, then give a proof of the negation.

   (a) $\forall x \in \mathbb{Q}$, x can be written as a/b where $a, b \in \mathbb{Z}$ and $b|a$.

   (b) $\exists x \in \mathbb{Q}$ so that x can be written as a/b where $a, b \in \mathbb{Z}$ and $b|a$.

   (c) $\forall x \in \mathbb{Q}$, x can be written as a/b where $a, b \in \mathbb{Z}$ and $b|a^2$.

   (d) $\forall x \in \mathbb{R}$, if x is irrational then x^2 is irrational or x^3 is irrational.

   (e) $\forall x \in \mathbb{R}$, if x^2 is rational then x^3 is rational or x^5 is rational.

2. (a) Disprove the following statement: $\forall x, y \in \mathbb{R}$, if $\lfloor xy \rfloor = 0$ then $\lfloor x \rfloor = 0$ or $\lfloor y \rfloor = 0$.

   (b) Write out the contrapositive of the statement in part (a). Is it true or false? Explain.

   (c) Write out the converse of the statement in part (a). Is it true or false? Explain.

   (d) Prove or disprove the following statement: $\forall x, y \in \mathbb{R}$, if $\lceil xy \rceil = 0$ then $\lceil x \rceil = 0$ or $\lceil y \rceil = 0$.

   (e) Prove or disprove the following statement: $\exists x, y \in \mathbb{R}$ such that $\lfloor xy \rfloor = 0$ and $\lfloor x \rfloor \lfloor y \rfloor = 271$.

   (f) Prove or disprove the following statement: $\exists x, y \in \mathbb{R}$ such that $\lceil xy \rceil = 0$ and $\lceil x \rceil \lceil y \rceil = 271$.

3. Let N be your student ID number.

   (a) **Use the Euclidean Algorithm** to find $\gcd(N, 271)$.

   (b) Use your answer to part (a) to write $\gcd(N, 271)$ in the form $Na + 271b$ where $a, b \in \mathbb{Z}$.

   (c) Suppose that M is a positive integer such that $\gcd(M, 271) = \gcd(M, 2008)$. Find $\gcd(M, 271)$. Explain. [*Hint*: 271 is prime.]

   (d) Suppose that M is an integer between 250000 and 450000 such that $\gcd(M, 271) = \gcd(M, 2008) + 20$. Find M. Explain. [You may use Exercise 33, page 631.]