UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

Quiz #3 - MATH 271 - L01
Thursday March 19, 2009

SOLUTIONS

Your family name: ______________________

Your first name: ______________________

Your signature: ______________________

Your student number: ______________________

INSTRUCTIONS:

I. Fill out the above information BEFORE starting.

II. **Show all your work**, use the back of the previous page for rough work and clearly insert your solution in the space provided space.

III. Calculators are not allowed, and no other material.

IV. There are 2 questions and 3 pages to this exam.

V. Time allowed is 50 minutes.

MARKS:	#1: /5	#2: /5	Total: /10

Question 1 (5 points)

[1] **a:** How many ways can ways can two integers be selected from the set $\{0, 1, 2, 3, \ldots, 100\}$?

the set contains 101 integers
so there are
$\binom{101}{2} = \frac{101 \cdot 100}{2} = 5050$ ways to select 2 integers.

[2] **b:** How many ways can ways can two integers be selected from the set $\{0, 1, 2, 3, \ldots, 100\}$ so that their sum is even?

Both must be even, or both odd.
there are 51 even integers in the set
50 odd - - - - -.

so
$\binom{51}{2} + \binom{50}{2} = \frac{51 \cdot 50}{2} + \frac{50 \cdot 49}{2} = 25(51+49) = 2500$ ways.
to have even sum.

[2] **c:** How many ways can ways can two integers be selected from the set $\{0, 1, 2, 3, \ldots, 100\}$ so that their sum is divisible by 3?

Both must be of form $3k$
or one of form $3k+1$ and other of form $3k+2$.

there are
34 integers in the set of the form $3k$ $(0, 3, \ldots, 99)$
34 - - - - - - - - $3k+1$ $(1, 4, \ldots, 100)$
33 - - - - - - - - - - $3k+2$ $(101 - 2 \cdot 34)$

so the number of ways is

$\binom{34}{2} + \binom{34}{1}\binom{33}{1} = \frac{34 \cdot 33}{2} + 34 \cdot 33 = \frac{3}{\cancel{2}} \times \overset{17}{\cancel{34}} \times 33 = 17 \cdot 99 =$
$= 1683$

Question 2 (5 points) Let m be any nonnegative integer. Use mathematical induction and Pascal's formula to prove that for all integers $n \geq 0$,

$$\binom{m}{0} + \binom{m+1}{1} + \cdots + \binom{m+n}{n} = \binom{m+n+1}{n}$$

Let $P(n)$ be the statement

$$\binom{m}{0} + \cdots + \binom{m+n}{n} = \binom{m+n+1}{n} \text{ for all } m.$$

We prove it by induction on $n \geq 0$

Base case $n=0$ We need to verify $P(0)$, i.e. $\binom{m}{0} = \binom{m+1}{0}$

But $\binom{m}{0} = \frac{m!}{0!\,m!} = 1$, $\binom{m+1}{0} = \frac{(m+1)!}{0!(m+1-0)!} = 1$

So $P(0)$ holds

Inductive Step Assume $P(n)$ is true for some $n \geq 0$

We need to prove $P(n+1)$

$$\binom{m}{0} + \cdots + \binom{m+n}{n} + \binom{m+n+1}{n+1}$$

$$= \binom{m+n+1}{n} + \binom{m+n+1}{n+1} \quad \text{[By inductive hyp]}$$

$$= \binom{m+(n+1)+1}{n+1} \quad \text{[by Pascal's Triangle]}$$

So $P(n+1)$ is true.

We conclude that $P(n)$ is true for all n.