UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

Quiz #4 - MATH 271 - L01

Thursday April 2, 2009

SOLUTIONS

Your family name: ______________________

Your first name: ______________________

Your signature: ______________________

Your student number: ______________________

INSTRUCTIONS:

I. Fill out the above information BEFORE starting.
II. **Show all your work**, use the back of the previous page for rough work and clearly insert your solution in the space provided space.
III. Calculators are not allowed, and no other material.
IV. There are 2 questions and 3 pages to this exam.
V. Time allowed is 50 minutes.

MARKS:	#1: /5	#2: /5	Total: /10

Question 1 (5 points) Let $S = \{1,2,3,4\}$ and $\mathcal{P}(S)$ the power set of S. A binary relation R on $\mathcal{P}(S)$ is defined as follows:

$$\text{For } X, Y \in \mathcal{P}(S),\ XRY \quad \leftrightarrow \quad N(X \cup Y) \text{ is odd.}$$

[1] **a:** With justification, determine whether the relation R is reflexive.

NO. For example. $\{1,2\} \not R \{1,2\}$

since $N(\{1,2\} \cup \{1,2\}) = N(\{1,2\}) = 2$ is even.

(Can use $\emptyset \not R \emptyset$ as well)

[2] **b:** With justification, determine whether the relation R is symmetric.

Yes if XRY, then $N(X \cup Y)$ is odd

But $X \cup Y = Y \cup X$

so $N(Y \cup X)$ is odd as well

so YRX by definition.

[2] **c:** With justification, determine whether the relation R is transitive.

No. For example

$X = \{1,2\}$ $Y = \{2,3\}$ $Z = \{3,4\}$

then $X \cup Y = \{1,2,3\}$ $N(X \cup Y)$ odd

$Y \cup Z = \{2,3,4\}$ $N(Y \cup Z)$ odd

$X \cup Y = \{1,2,3,4\}$ $N(X \cup Y)$ even

so $XRY \wedge YRZ$

but $X \not R Z$.

Question 2 (5 points) Let $f : X \to Y$ be a function. Define a binary relation on X by:

$$x_1 R x_2 \;\leftrightarrow\; f(x_1) = f(x_2).$$

[3] **a:** With justification, show that R is an equivalence relation on X.

① xRx since $f(x)=f(x)$.

② ~~If~~ xRy then $f(x)=f(y)$

so $f(y)=f(x)$, ie yRx.

③ if xRy ∧ yRz

then $f(x)=f(y)$ ∧ $f(y)=f(z)$

so $f(x)=f(z)$

ie xRz.

[2] **b:** Describe the equivalence classes when f is one-to-one.

For ~~all~~ $x \in X$

$[x] = \{y : xRy\}$

$= \{y : f(x)=f(y)\}$

$= \{y : x=y\}$ since f is one-one.

$= \{x\}$

So $[x] = \{x\}$ for each x.

UNIVERSITY OF
CALGARY

Faculty of Science
Department of Mathematics & Statistics

Quiz #4 - MATH 271 - L02
Thursday April 2, 2009

Your family name: ______________________

Your first name: ______________________

Your signature: ______________________

Your student number: ______________________

INSTRUCTIONS:
I. Fill out the above information BEFORE starting.
II. **Show all your work**, use the back of the previous page for rough work and clearly insert your solution in the space provided space.
III. Calculators are not allowed, and no other material.
IV. There are 2 questions and 3 pages to this exam.
V. Time allowed is 50 minutes.

MARKS:	#1: /5	#2: /5	Total: /10

Question 1 (5 points) Let $S = \{1, 2, 3, 4\}$ and $\mathcal{P}(S)$ the power set of S. A binary relation R on $\mathcal{P}(S)$ is defined as follows:

$$\text{For } X, Y \in \mathcal{P}(S),\ \ XRY \ \leftrightarrow\ N(X \cap Y) \text{ is even.}$$

[1] **a:** With justification, determine whether the relation R is reflexive.

No For example Let $X=\{1\}$

then $X \cap X = X = \{1\}$

so $N(X \cap X)$ is odd.

i.e. $X \not R X$.

[2] **b:** With justification, determine whether the relation R is symmetric.

Yes. Suppose XRY, i.e. $N(X \cap Y)$ is even.

But $X \cap Y = Y \cap X$ so $N(Y \cap X)$ is even

i.e. YRX.

[2] **c:** With justification, determine whether the relation R is transitive.

No, For example,

Let $X = \{1\}$ $Y = \{2\}$ $Z = \{1,3\}$

& $X \cap Y = \emptyset = Y \cap Z$ so $N(X \cap Y) = 0 = N(Y \cap Z)$ is even.

i.e. $XRY \wedge YRZ$

But $X \cap Z = \{1\}$ i.e. $N(X \cap Z)$ is odd

$X \not R Z$.

Question 2 (5 points) Let R be an equivalence relation on a set A.

[1] **a:** For $a \in A$, define what is the equivalence class $[a]$.

$[a] = \{b : aRb\}$

[4] **b:** For $a, b \in A$, prove that $[a] \cap [b] \neq \emptyset$ implies that $[a] = [b]$.

Let $x \in [a] \cap [b]$. We prove $[a] = [b]$
So $aRx \wedge bRx$.

a) $[a] \subseteq [b]$
Let $y \in [a]$ ie aRy
then $bRx \wedge xRa \wedge aRy$ (symmetry)
$\therefore bRy$ (transitivity twice)
$\therefore y \in [b]$

b) $[b] \subseteq [a]$
Let $y \in [b]$ ie bRy
then $aRx \wedge xRb \wedge bRy$ (symmetry)
$\therefore aRy$ (transitivity twice)
$\therefore y \in [a]$

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