UNIVERSITY OF CALGARY

Faculty of Science
Department of Mathematics & Statistics

Homework #1 - MATH 271 - L01 & L02

Follow instructions available in the Assignment Policy document!

Question 1 For each true statement below, give a proof. For each false statement below, write out its negation, then give a proof of the negation.

a: $(\forall x, y \in \mathbb{R}^+)\ \lfloor xy \rfloor = \lfloor x \rfloor \lfloor y \rfloor$

b: $(\forall x, y \in \mathbb{R}^+)$ if $y \geq 1$ then $\left\lfloor \frac{x}{y} \right\rfloor = \left\lfloor \frac{\lfloor x \rfloor}{\lfloor y \rfloor} \right\rfloor$

c: $(\forall x \in \mathbb{R})(\forall n \in \mathbb{N})$ if $x - \lfloor x \rfloor < \frac{1}{n}$ then $\lfloor nx \rfloor = n \lfloor x \rfloor$

d: $(\forall n \in \mathbb{Z})\ \lceil \frac{n^2}{4} \rceil = \lceil \frac{n^2+3}{4} \rceil$ if and only if n is odd.

Question 2 For each true statement below, give a proof. For each false statement below, write out its negation, then give a proof of the negation.

a: The product of irrational numbers is irrational.

b: $\sqrt{6}$ is irrational.

Question 3 For each true statement below, give a proof. For each false statement below, write out its negation, then give a proof of the negation.

a: $(\forall a, b \in \mathbb{Z})(a \neq 0 \vee b \neq 0)$ implies $gcd(a, b) = gcd(a, a - b)$.

b: $(\forall a, b \in \mathbb{Z})(a \neq 0 \vee b \neq 0)$ implies $gcd(a, b) = gcd(a + b, a - b)$.